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Modulation and parsvals theorem of Two Dimensional offset Fractional Fourier Transform

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Abstract: The Fractional Fourier transform is generalization of conventional Fourier transform. It is most important tool for two dimensional signal processing. In resent year, the fractional Fourier transform has been investigated by many different authors and they proved, FRFT is very useful in solving some problem in quantum physics, optics, filter design, image processing, pattern recognition, wavelet transform.

The Offset fractional Fourier transform is the space shifted frequency modulated version of original one. In this paper we have presented Generalization of Two Dimensional Offset Fractional Fourier transform (2D offset FRFT). Modulation and parsvals theorem for the Two dimensional Offset fractional Fourier transform are also proved.

Keywords: Fourier Transform; Fractional Fourier Transform; Two Dimensional offset Fractional Fourier transform; Generalized Function; Testing function space.

I. INTRODUCTION

Fractional Fourier transform is time-frequency distribution which is extensively used in signal processing community. It was first introduced by Namias in 1980. Namias applied his technique to the free and the forced quantum mechanical harmonic oscillator [1]. His results were later modified by McBride and Kerr who also developed among other investigations an operational calculus for the fractional Fourier transform [3].

Fractional Fourier transform has the energy preservation property because the FRFT is base on the decomposition of the signal on the orthonormal basis set of the chirp functions. This property can obtain by the application of Parseval s theorem. Due to the energy preserving property of the Fourier Transform, the squared magnitude of the Fourier transform of signal is often called the energy spectrum of the signal and is interpreted as the distribution of the signal energy among the different frequencies [4].

In the area of facial expression recognition, some researchers have shown its superiority with respect to other feature extraction tool [6]. Two dimensional fractional Fourier transform is powerful tool for feature extraction in facial expression recognition. It knows that different order of two dimensional fractional Fourier transform contain different time frequency information. Two-Dimensional Offset Fractional Fourier transform is the extension of Two Dimensional Fractional Fourier transform. Offset Fractional Fourier transform is useful in optics. They are especially useful for analyzing optical system with prises or shifted lenses [7].

A. Two Dimensional Offset Fractional Fourier Transform:

Two Dimensional Offset Fractional Fourier Transform $\left[F_\alpha^{\tau,\eta,\zeta,\gamma} f(t,x)\right](s,u)$ of function f (t, x) through an angle α *is* defined as

$$[F_\alpha^{\tau,\eta,\zeta,\gamma} f(t,x)](s,u) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t,x) k_\alpha(t,s-\eta,x,u-\gamma).dt.dx$$

Where, $$k_\alpha(t,s-\eta,x,u-\gamma) = \sqrt{\frac{1-i\cot\alpha}{2\pi}} e^{i(s\tau+u\zeta)} e^{\frac{1}{2\sin\alpha}[((s-\eta)^2+t^2+(u-\gamma)^2+x^2)\cos\alpha-2((s-\eta)t+(u-\gamma)x)]}$$

B. Testing function space E:

An infinitely deferential complex valued smooth function on $\phi(R^n)$ belongs to (R^n) , if for each contact $I \subset S_{a,b}$

Where,

$$S_{a,b} = \{t;x:t\,x \in R^n\}\ ;\quad |t| \le a, |x| \le b, a < 0, b < 0\}, I \in R^n$$

$$\gamma_{I,l,q}(\phi) = \sup_{t,x,\in I} |D_{t,x}^{l,q}\phi(t,x)|$$

$$< \infty, \qquad l,q = 0,1,2 \ldots\ldots\ldots$$

In the present paper we have proposed Modulation and parsvels theorem for two- dimensional Offset fractional Fourier transform.

Distributional Two-Dimensional Offset Fractional Fourier Transform

The Two-Dimensional Offset Fractional Fourier Transform $[F_\alpha^{\tau,\eta,\zeta,\gamma} f(t,x)](s,u)$ of generalization function $f\,(t,x)$ through an angle α is defind as, $[F_\alpha^{\tau,\eta,\zeta,\gamma} f(t,x)](s,u) = \langle f(t,x)\ K_\alpha(t,s-\eta,x,u-\gamma)\rangle$

Where $$K_\alpha(t,s-\eta,x,u-\gamma) = C_{1\alpha} e^{i(s\tau+u\zeta)} e^{C_{2\alpha}[((s-\eta)^2+t^2+(u-\gamma)^2+x^2)\cos\alpha-2((s-\eta)t+(u-\gamma)x)]}$$

Where $C_{1\alpha} = \sqrt{\frac{1-i\cot\alpha}{2\pi}}$ and $C_{2\alpha} = \frac{1}{2\sin\alpha}$

II. Modulation Theorem of 2D Offset Fractional Fourier Transform

A. Theorem I

If $\{2D\ offset\ FRFT\ f(t,x)\}(s,u)$ denotes generalized two-dimensional offset FRFT of $f(t,x)$ then

$\{2D\ offset\ FRFT f(t,x)\cos[at+bx]\}(s,u)$

$$= \frac{C_{1\alpha}}{2} e^{\frac{i}{2}\cos\alpha\sin\alpha(a^2+b^2)} \Big\{\{2FRFT\ f(t,x)e^{i[(s-\eta)a+(u-\gamma)b]\cos\alpha}\}((s-\eta)-a\sin\alpha)\,((u-\gamma)-b\sin\alpha)$$
$$+\{2D\ offset\ FRFT\ f(t,x)e^{-i[(s-\eta)a+(u-\gamma)b]\cos\alpha}\}((s-\eta)+a\sin\alpha)\,((u-\gamma)+b\sin\alpha)\Big\}$$

Proof:

$\{2Doffset\ FRFT f(t,x)\cos[at+bx]\}(s,u)$

$$= C_{1\alpha} e^{i(s\tau+u\xi)} e^{\frac{i}{2}[(s-\eta)^2+(u-\gamma)^2]\cot\alpha} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{\frac{i}{2}[t^2+x^2]\cot\alpha-i[t(s-\eta)+(u-\gamma)x]\mathrm{cosec}\,\alpha} - \cos[at+bx]\ f(t,x)dt\,d$$

$$= C_{1\alpha} e^{i(s\tau+u\xi)} e^{\frac{i}{2}[(s-\eta)^2+(u-\gamma)^2]\cot\alpha} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{\frac{i}{2}[t^2+x^2]\cot\alpha-i[t(s-\eta)+(u-\gamma)x]\mathrm{cosec}\,\alpha} \left[\frac{e^{(at+bx)i}+e^{-(at+bx)i}}{2}\right] f(t,x)dt\,dx$$

$$= \frac{C_{1\alpha}}{2} e^{i(s\tau+u\xi)} e^{\frac{i}{2}[(s-\eta)^2+(u-\gamma)^2]\cot\alpha} \left\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{\frac{i}{2}[t^2+x^2]\cot\alpha-i[t(s-\eta)+(u-\gamma)x-(at+bx)\sin\alpha]\mathrm{cosec}\,\alpha} f(t,x)dt\,dx\right.$$

$$+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{\frac{i}{2}[t^2+x^2]\cot\alpha - i\,\mathrm{cosec}\,\alpha[t(s-\eta)+(u-\gamma)x-(at+bx)\sin\alpha]}\ f(t,x)dt\,dx\Big\}$$

$$=\frac{C_{1\alpha}}{2}e^{i(s\tau+u\xi)}\Big\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{\frac{i}{2}[[(s-\eta)-a\sin\alpha]^2+[(u-\gamma)-b\sin\alpha]^2]\cot\alpha} e^{i[(s-\eta)a+(u-\gamma)b]\cos\alpha}$$

$$e^{-\frac{i}{2}[a^2+b^2]\sin\alpha\cos\alpha}\ e^{\frac{i}{2}\cdot(t^2+x^2)\cot\alpha - i\,\mathrm{cosec}\,\alpha[s-\eta-a\sin\alpha]t - i\,\mathrm{cosec}\,\alpha[u-\gamma-b\sin\alpha]x} f(t,x)dt\,dx$$

$$+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{\frac{i}{2}[[(s-\eta)-a\sin\alpha]^2+[(u-\gamma)-b\sin\alpha]^2]\cot\alpha} e^{-i[(s-\eta)a+(u-\gamma)b]\cos\alpha}$$

$$e^{-\frac{i}{2}[a^2+b^2]\sin\alpha\cos\alpha}\ e^{\frac{i}{2}\cdot(t^2+x^2)\cot\alpha - i\,\mathrm{cosec}\,\alpha[(s-\eta)^2+a\sin\alpha]t - i\,\mathrm{cosec}\,\alpha[(u-\gamma)+b\sin\alpha]x} f(t,x)dt\,dx\Big\}$$

$$=\frac{C_{1\alpha}}{2}e^{-\frac{i}{2}[a^2+b^2]\sin\alpha\cos\alpha}\Big\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{i(s\tau+u\xi)} e^{\frac{i}{2}\cdot[(t^2+x^2)+[s-\eta-a\sin\alpha]^2+[(u-\gamma)+b\sin\alpha]^2]\cot\alpha}$$

$$e^{it\,\mathrm{cosec}\,\alpha[s-\eta-a\sin\alpha]-ix\,\mathrm{cosec}\,\alpha\,[(u-\gamma)+b\sin\alpha]} f(t,x) e^{icos\,\alpha[a(s-\eta)+b(u-\gamma)]} f(t,x)dt\,dx$$

$$+\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{i(s\tau+u\xi)} e^{\frac{i}{2}\cdot[(t^2+x^2)+[s-\eta-a\sin\alpha]^2+[(u-\gamma)+b\sin\alpha]^2]\cot\alpha}$$

$$e^{-it\,\mathrm{cosec}\,\alpha[s-\eta+a\sin\alpha]-ix\,\mathrm{cosec}\ [(u-\gamma)+b\sin\alpha]} f(t,x) e^{-i\,cos\,\alpha[a(s-\eta)+b(u-\gamma)]}dt\,dx$$

$$=\frac{C_{1\alpha}}{2}e^{\frac{i}{2}\cos\alpha\sin\alpha(a^2+b^2)}\Big\{\{2DoffsetFRFT\ f(t,x)e^{i[(s-\eta)a+(u-\gamma)b]\cos\alpha}\}((s-\eta)-\mathrm{a}\sin\alpha)\ \big((u-\gamma)-b\sin\alpha\big)$$

$$+\{2D\ offset\ FRFT\ f(t,x)e^{-i[(s-\eta)a+(u-\gamma)b]\cos\alpha}\}((s-\eta)-\mathrm{a}\sin\alpha)\ ((u-\gamma)-b\sin\alpha)\Big\}$$

Hence Proved

B. Theorem II

If $\{2D\ offset\ FRFT\ f(t,x)\}(s,u)$ denotes generalized two-dimensional offset FRFT of $f(t,x)$ then

$$\{2D\ offset\ FRFT\ f(t,x)\sin[at+bx]\}(s,u)$$

$$=-i\frac{C_{1\alpha}}{2}e^{\frac{i}{2}\cos\alpha\sin\alpha(a^2+b^2)}\Big\{\{2D\ offset\ FRFT\ f(t,x)e^{i[(s-\eta)a+(u-\gamma)b]\cos\alpha}\}((s-\eta)-\mathrm{a}\sin\alpha)\ \big((u-\gamma)-b\sin\alpha\big)$$

$$+\{2D\ offset\ FRFT\ f(t,x)e^{-i[(s-\eta)a+(u-\gamma)b]\cos\alpha}\}((s-\eta)+\mathrm{a}\sin\alpha)\ ((u-\gamma)+b\sin\alpha)\Big\}$$

Proof:

$$\{2Doffset\ FRFTf(t,x)\cos[at+bx]\}(s,u)$$

$$=C_{1\alpha}e^{i(s\tau+u\xi)}e^{\frac{i}{2}[(s-\eta)^2+(u-\gamma)^2]\cot\alpha}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{\frac{i}{2}[t^2+x^2]\cot\alpha - i[t(s-\eta)+x(u-\gamma)]\mathrm{cosec}\,\alpha}\ .\sin[at+bx]\ f(t,x)dt\,dx$$

$$=C_{1\alpha}e^{i(s\tau+u\xi)}e^{\ \frac{i}{2}[(s-\eta)^2+(u-\gamma)^2]\cot\alpha}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{\frac{i}{2}[t^2+x^2]\cot\alpha - i[t(s-\eta)+x(u-\gamma)]\mathrm{cosec}\,\alpha}.\left[\frac{e^{(at+bx)i}+e^{-(at+bx)i}}{2i}\right] f(t,x)dt\,dx$$

$$=\frac{-iC_{1\alpha}}{2}e^{i(s\tau+u\xi)}e^{\frac{i}{2}[(s-\eta)^2+(u-\gamma)^2]\cot\alpha}\Big\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{\frac{i}{2}[t^2+x^2]\cot\alpha - i[t(s-\eta)+x(u-\gamma)-(at+bx)\sin\alpha]\mathrm{cosec}\,\alpha} f(t,x)dt\,dx$$

$$-\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{\frac{i}{2}[t^2+x^2]\cot\alpha - i\,\mathrm{cosec}\,\alpha[t(s-\eta)+(u-\gamma)x-(at+bx)\sin\alpha]} f(t,x)dt\,dx\Big\}$$

$$=\frac{-iC_{1\alpha}}{2}e^{i(s\tau+u\xi)}\Big\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{\frac{i}{2}[[(s-\eta)-a\sin\alpha]^2+[(u-\gamma)-b\sin\alpha]^2]\cot\alpha} e^{i[(s-\eta)a+(u-\gamma)b]\cos\alpha}$$

$$e^{-\frac{i}{2}[a^2+b^2]\sin\alpha\cos\alpha} e^{\frac{i}{2}(t^2+x^2)\cot\alpha - i\,\mathrm{cosec}\,\alpha[s-\eta-a\sin\alpha]t - i\,\mathrm{cosec}\,\alpha[u-\gamma-b\sin\alpha]x} f(t,x)dt\,dx$$

$$-\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{\frac{i}{2}[[(s-\eta)-a\sin\alpha]^2+[(u-\gamma)-a\sin\alpha]^2]\cot\alpha} e^{-i[(s-\eta)a+(u-\gamma)b]\cos\alpha} e^{-\frac{i}{2}[a^2+b^2]\sin\alpha\cos\alpha}$$

$$e^{\frac{i}{2}(t^2+x^2)\cot\alpha - i\,\mathrm{cosec}\,\alpha[(s-\eta)^2+a\sin\alpha]t - i\,\mathrm{cosec}\,\alpha[(u-\gamma)+b\sin\alpha]x} f(t,x)dt\,dx\Big\}$$

$$= \frac{-iC_{1\alpha}}{2} e^{-\frac{i}{2}[a^2+b^2]\sin\alpha\cos\alpha}\Big\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{i(s\tau+u\xi)} e^{\frac{i}{2}(t^2+x^2)\cot\alpha + [[s-\eta-a\sin\alpha]^2+[(u-\gamma)+b\sin\alpha]^2]\cot\alpha}$$

$$e^{it\,\mathrm{cosec}\,\alpha[s-\eta-a\sin\alpha] - ix\,\mathrm{cosec}\,\alpha\,[(u-\gamma)+b\sin\alpha]} f(t,x) e^{i\cos\alpha[a(s-\eta)+b(u-\gamma)]} dt\,dx$$

$$-\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{i(s\tau+u\xi)} e^{\frac{i}{2}(t^2+x^2)+[[s-\eta-a\sin\alpha]^2+[(u-\gamma)+b\sin\alpha]^2]\cot\alpha}$$

$$e^{-it\,\mathrm{cosec}\,\alpha[s-\eta+a\sin\alpha] - ix\,\mathrm{cosec}\,\alpha\,[(u-\gamma)+b\sin\alpha]}\ f(t,x) e^{-i\cos\alpha[a(s-\eta)+b(u-\gamma)]} dt\,dx$$

$$= \frac{-iC_{1\alpha}}{2} e^{-\frac{i}{2}(a^2+b^2)\cos\alpha\sin\alpha}\Big\{\big\{2FRFT\ f(t,x)e^{i[(s-\eta)a+(u-\gamma)b]\cos\alpha}\big\}(s-\eta-\mathrm{a}\sin\alpha)\,(u-\gamma-b\sin\alpha)$$

$$+\big\{2D\ offset\ FRFT\ f(t,x)e^{-i[(s-\eta)a+(u-\gamma)b]\cos\alpha}\big\}(s-\eta-\mathrm{a}\sin\alpha)\,(u-\gamma-b\sin\alpha)\Big\}$$

Hence Proved

III. Parseval's Identity for 2D Offset FRFT

If $\{2D\ offset\ FRFT\ f(t,x)\} = F_\alpha^{\tau,\eta,\zeta,\gamma}(s,u)$ And $\{2D\ offset\ FRFT\ g(t,x)\} = G_\alpha^{\tau,\eta,\zeta,\gamma}(s,u)$ then

(A) $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t,x)\overline{g(t,x)}\ dt\,dx = A\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F_\alpha^{\tau,\eta,\zeta,\gamma}(s,u)\ \overline{G_\alpha^{\tau,\eta,\zeta,\gamma}(s,u)}\,dt\,dx$ where $A = \frac{1}{2\pi\sin\alpha}$

(B) $\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}|f(t,x)|^2\ dt\,dx = A\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left|F_\alpha^{\tau,\eta,\zeta,\gamma}(s,u)\right|^2\ dt\,dx$

Proof:

A. *By definition of* $2D\ offset\ FRFT$

$$\{2D\ offset\ FRFT\ f(t,x)\}(s,u) = F_\alpha^{\tau,\eta,\zeta,\gamma}(s,u)$$

$$= C_{1\alpha} e^{\frac{i}{2}[(s-\eta)^2+(u-\gamma)^2]\cot\alpha}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{i(s\tau+u\xi)}\ e^{\frac{i}{2}[t^2+x^2]\cot\alpha - i[(s-\eta)+t+(u-\gamma)x]\,\mathrm{cosec}\,\alpha} f(t,x)dx\,dy$$

By using Inversion Formula of $2D\ offset\ FRFT$

$$g(t,x)$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}[G_\alpha^{\tau,\eta,\zeta,\gamma} g(t,x)](s,u)\frac{1}{2\pi^{\frac{3}{2}}}\frac{1}{\sin^2\alpha}\frac{1}{\sqrt{1-i\cot\alpha}} e^{-i(s\tau+u\xi)}\ e^{\frac{-i}{2\sin\alpha}\left[[(s-\eta)^2+t^2+(u-\gamma)^2+x^2]\cos\alpha\right]-2t(s-\eta)-2(u-\gamma)x} ds\,du$$

$$\overline{g(t,x)} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\overline{G_\alpha^{\tau,\eta,\zeta,\gamma}}(s,u)\frac{1}{2\pi^{\frac{3}{2}}}\frac{1}{\sin^2\alpha}\frac{1}{\sqrt{1-i\cot\alpha}}\ e^{i(s\tau+u\xi)}\ e^{\frac{i}{2\sin\alpha}\left[[(s-\eta)^2+t^2+(u-\gamma)^2+x^2]\cos\alpha\right]-2(s-\eta)t-2(u-\gamma)x} ds\,du$$

Now $$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} f(t,x)\overline{g(t,x)}\,dt\,dx = \frac{1}{2\pi\sqrt{1+i\cot\alpha}}\frac{1}{\sqrt{1-i\cot\alpha}}\frac{1}{\sin^2\alpha}$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\overline{G_{\alpha}^{\tau,\eta,\zeta,\gamma}}(s,u)\,ds\,du\left\{\sqrt{\frac{1-i\cot\alpha}{2\pi}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}e^{i(s\tau+u\xi)}\,e^{\frac{1}{2\sin\alpha}\left[\left[(s-\eta)^2+t^2+(u-\gamma)^2+x^2\right]\cos\alpha\right]-2[(s-\eta)+t+(u-\gamma)x]}f(t,x)dt\,dx\right\}$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}f(t,x)\overline{g(t,x)}\,dt\,dx=\frac{1}{2\pi\sin\alpha}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\overline{G_{\alpha}^{\tau,\eta,\zeta,\gamma}}(s,u)F_{\alpha}^{\tau,\eta,\zeta,\gamma}(s,u)\,ds\,du$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}f(t,x)\overline{g(t,x)}\,dt\,dx=A\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\overline{G_{\alpha}^{\tau,\eta,\zeta,\gamma}}(s,u)F_{\alpha}^{\tau,\eta,\zeta,\gamma}(s,u)\,ds\,du$$

Where $A=\frac{1}{2\pi\sin\alpha}$

B. *Let* $f(t,x)=g(t,x)$ *then* $F_{\alpha}^{\tau,\eta,\zeta,\gamma}(s,u)=G_{\alpha}^{\tau,\eta,\zeta,\gamma}(s,u)$

And $\overline{F_{\alpha}^{\tau,\eta,\zeta,\gamma}(s,u)}=\overline{G_{\alpha}^{\tau,\eta,\zeta,\gamma}(s,u)}$

Now

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}f(t,x)\overline{f(t,x)}\,dt\,dx=A\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}F_{\alpha}^{\tau,\eta,\zeta,\gamma}(s,u)\,\overline{F_{\alpha}^{\tau,\eta,\zeta,\gamma}(s,u)}\,ds\,du$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}|f(t,x)|^2\,dt\,dx=A\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\left|F_{\alpha}^{\tau,\eta,\zeta,\gamma}(s,u)\right|^2\,ds\,du$$

Hence Proved

IV. Conclusion

In this paper we have defined Modulation and parsvels theorem for two- dimensional Offset fractional Fourier transform are proved.

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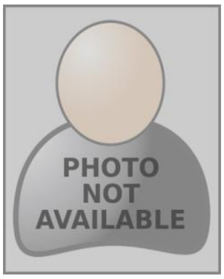

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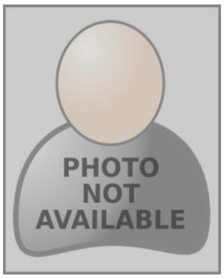

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