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Mathematical modeling of Social Networks: Reliability and Security of relationship among different nodes in Social Networks

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Abstract: Srivastav et al already published a paper where they have established a graph theoretical model to describe social networks. They have shown that it is possible to describe any social networks using simple nodes and edges. In the present work the authors have made further study on mathematical modeling of Social area networks. In social networks the senders send their messages to one or many persons. It basically produces relationship among different users. The authors have studied mathematically the relationship among the users. Normally when a person likes another person then the relationship is reliable and the messages remain secured. But a person starts to dislike another person then there is no reliability and the messages are no more secured. In the present study the authors tried to explore this concept in a systematic manner.

Keywords: social networks; reliability; security; private message; relationship.

I. INTRODUCTION

The fundamental thing of social networking is to connect the nodes of social networks and send the message from one node to another node at any time. Social networking is the interrelationship between the object / nodes. The practical approach of social network is to make either connection between two nodes or make disconnection over a period of time after bearing the relationship. Therefore, the fundamental question that arises here is that how much two or more relationship in social networking will be reliable and also the security of relationship on the basis of leak of messages (conversation) send by these nodes in between themselves. Reliability is the probability that a device or system will perform its intended function for a given interval of time under specified operating system.

Security is the state of being free from danger or threat. Message send from one node to another may be public or private. So, there arise a question related to message security. Message security is the practice of encrypting messages on our device so that they can be read only by the intended recipient. In the private, message send from one node to another node is either done Confidentially or Authentically. In case of confidential message, the message encryption is the only way to ensure that only the indented recipients are reading our message and in authenticity message encryption is the only way to ensure the identity of the people we are communicating with. Thus, the authentic nodes have the collection of private message send by the another nodes. Hence, the Reliability and Security of relationship between two nodes depend on the situation of connection or disconnection of nodes.

II. MATHEMATICAL DESCRIPTION OF RELATIONSHIP BETWEEN TWO NODES

A social network is a social structure made up of individuals (or organizations) called "nodes", which are tied (connected) by one or more specific types of interdependency, such as friendship, kinship, common interest, financial exchange, likings or disliking, or relationships of beliefs, knowledge or prestige. Social network analysis views social relationships in terms of

network theory consisting of nodes and ties (also called edges, links, or connections). Nodes are the individual actors within the networks, and ties are the relationships between the actors.

Hence, mathematically, social networking can be defined as the collection of socially connected elements/objects. i.e set

$$S = \{ \text{social elements} : \text{social elements are connected} \}$$

In a social network S, consider two nodes (i) node1 represented by N_1 and (ii) node2 represented by N_2 and consider a function f defined between node N_1 and node N_2

$$\text{i.e., } f: N_1 \rightarrow N_2$$

such that message send by node N_1 is received by node N_2

i.e, if $m_1, m_2, m_3, \dots, m_n$ be the message send by the node N_1 to N_2 , then $f(m_1), f(m_2), f(m_3), \dots, f(m_n)$ will be message received by N_2 .

$$\text{i.e., } \{m_1, m_2, m_3, \dots, m_n\} \rightarrow \{f(m_1), f(m_2), f(m_3), \dots, f(m_n)\}$$

i.e, set of send message by node $N_1 \rightarrow$ set of received message by node N_2 .

2.1 Reliability and security of relationship of nodes

In the social networking, there is need of connection to establish relationship between nodes of social network.



Figure-1

To establish this relationship there may exist different types of cases :

Case(i): Nodes N_1 may send friend request to nodes N_2 . Nodes N_2 has option to accept or reject friends request of node N_1 .

SENDER	RECEIVER
REQUEST MADE FOR CONNETION	REQUEST ACCEPTED
	REQUEST REJECTED

Table1

The following situation may occur during the above situation:

SENDER	RECIEVER
RELIABLE TO SEND	SECURE TO ACCEPT
	NOT SECURE TO ACCEPT

Table: 2

The following diagram shows the request make to connect and send message in social network:

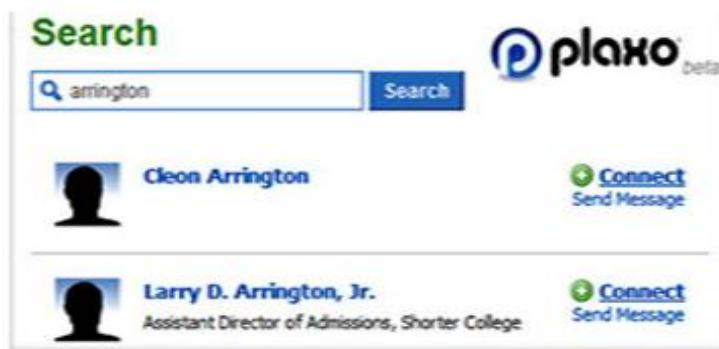


Figure-2

Case(ii): When nodes N_1 sends message to nodes N_2 after friendship(connection) then nodes N_2 has option to like or dislike the message of node N_1 .

SENDER	RECIEVER
MESSAGE SEND BY THE SENDER	MESSAGE LIKED BY THE RECIEVER
	MESSAGE DISLIKED BY THE RECIEVER

Table -3

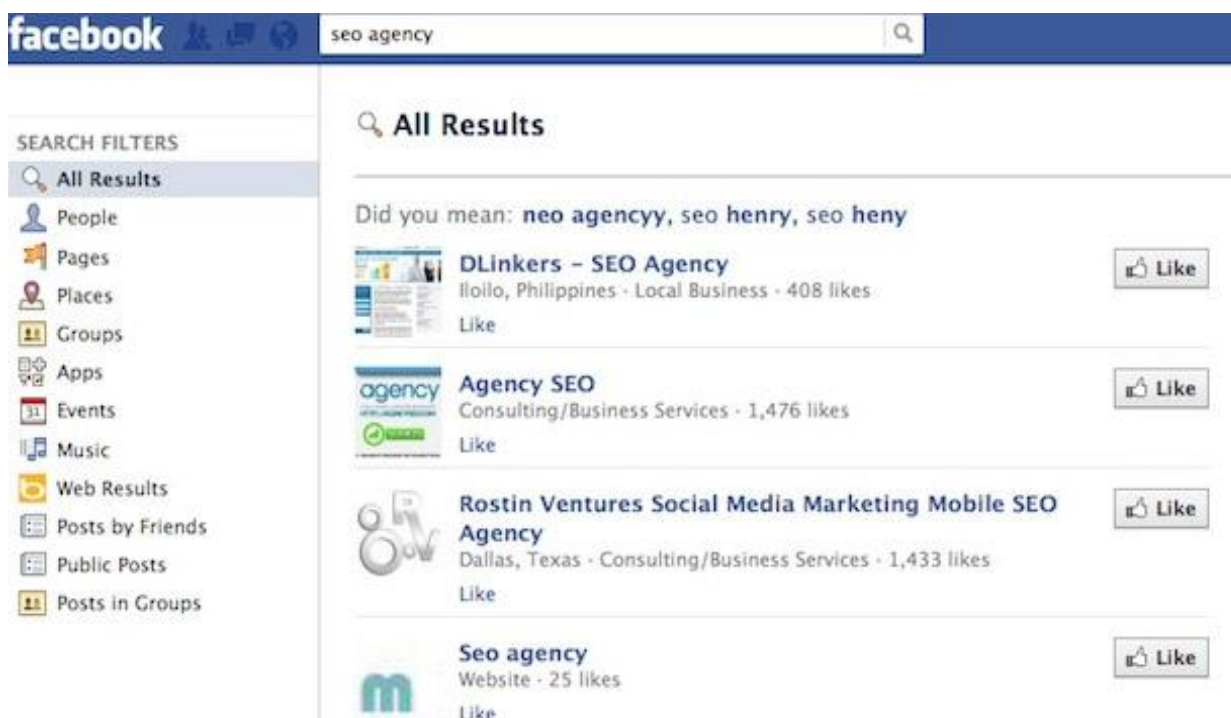


Figure-3

The reliability and security of like or dislike a message can be as follows:

SENDER	RECIEVER
MESSAGE SEND BY THE SENDER IS RELIABLE	MESSAGE LIKED BY THE RECIEVER IS SECURED
	MESSAGE DISLIKED BY THE RECIEVER IS NOT SECURED

Table:4

2.2 Sharing Of Message and Series and Parallel Relation between Nodes

(i) **One to One** - Sharing of message from one node to another node can be done using switching circuit network method in one to one way as follows:

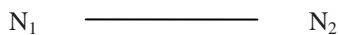


Figure : 4(a)

Hence, this type of relation may be extended(considered) as series relation which can be described as follows: considering the nodes N₁ , N₂, N₃, N₄ are connected as shown below:



Figure : 4(b)

In the above relation, nodes N₄ can not receive the message send by nodes N₁ directly .Nodes N₄ has to depend on the node N₁.Hence, the reliability and security of relationship between nodes N₁ and N₄ will always depends on the node N₂ and node N₃.

(ii) **One to Many**- Sharing of message from one node to another node can be done using switching circuit network method in one to many way as follows :

Let us considered nodes N₁,N₂,N₃,N₄,.....,N_n in the following diagram :

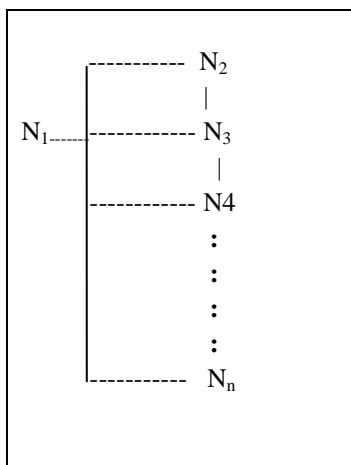


Figure:5

Here, node N₁ can share any message to simultaneously to all nodes N₂,N₃,N₄,.....,N_n .This type of connection is called parallel relation . Hence, the reliability and security of nodes N₂,N₃,N₄,.....,N_n do not depends on each other. Its only depend on node N₁.

2.3 Synthesis of Contact Networks

CONTACT NETWORKS:

A Contact is a bilateral device . A contact network is a network of interconnected contacts.

(i) **Single contact Network** : A Contact network in which every binary variable x_i (either in un-complemented or complemented form) is associated with only one edge is called a single-contact(or SC) network. Thus each contact in SC(single contact) network can be opened or closed independently.

(ii) **Realization of an SC (single contact) function** : Every path between nodes N₁ and nodes N₂ in the network are intended to design.

2.4. Variational Relation between Reliability and Security of Nodes

The idea of security and reliability are technically have grown up in different domains of thinking. In one context we may get a feeling of security because relationship is reliable. In other words we may feel that our relationship is reliable because it is secured. These are qualitative assumptions.

Security can be defined as a functional statistical statement of predictability where the essential question of being secure or not is whether a given system specified can be expected to continue to function for some period in some specifiable manner.

Reliability can be defined as a functional statistical statement of predictability where the essential question of being in or out of a reliable state is whether a given system specified can be expected to continue to function for some specifiable period in some specifiable manner.

The "reliability of security" is often considered but the "security of reliability" is not often considered. A prediction of reliability must assume that the system will not be altered or affected by external forces which are not proscribed in the specification during the expected period of performance. Reliability assumes a degree of security which assures that there is a criteria difference between normal and abnormal functionality.

When the nodes is connected to a network the reliability and security criteria change. The statement of connectivity is therefore also part of the system specification of security and reliability because all systems involve functionality in the presence of external influences.

2.4.1 Reliability between Two Nodes

Reliability is the probability of performing without failure, a specific function under given conditions for a specified period of time. The message send from one nodes to another node will be reliable until one node does not get harm/deceive from another nodes .Generally, one nodes can be reliable to another nodes by knowing the characteristics of another nodes. In any case nodes, nodes will be reliable on itself.

2.4.2 Security between Two Nodes

Security is major concerns for the nodes of social network .Nodes should always be alert to avoid false friendship. The relationship between two nodes of social network may be either personal or professional. In the social network message sent from one nodes to another nodes may be of the following type.

- (i)Public (ii)Private.

Therefore, before connection from one node to another node the security condition for the nodes should be checked.

The following diagram shows some case of security to connect nodes in social network in which nodes can send message either from one nodes to one node or one nodes to many nodes. The nodes can show every information to single node or every nodes of social network. To make a relation secure, a node has always an option to choose the relationship with another nodes either publically or privately at each stage.

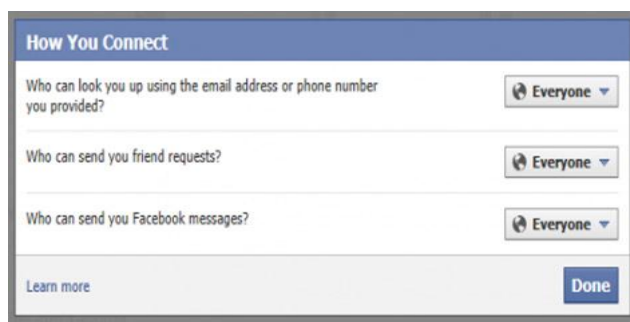


Figure:6

2.4.3 Graphical Representation of Relationship between Reliability and Security of Nodes

Let R denotes the reliability of two nodes of social network and S denotes the security between two nodes of social network, then it can be measured in the following cases:

Case1: When two nodes are connected and share the message in the same social network.

In this case **R varies directly with S during connection.**

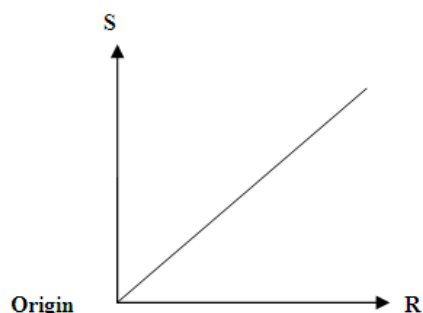


Figure: 7

Case2: When two nodes are disconnected after friendship.

In this case, **R varies inversely with S during disconnection.**

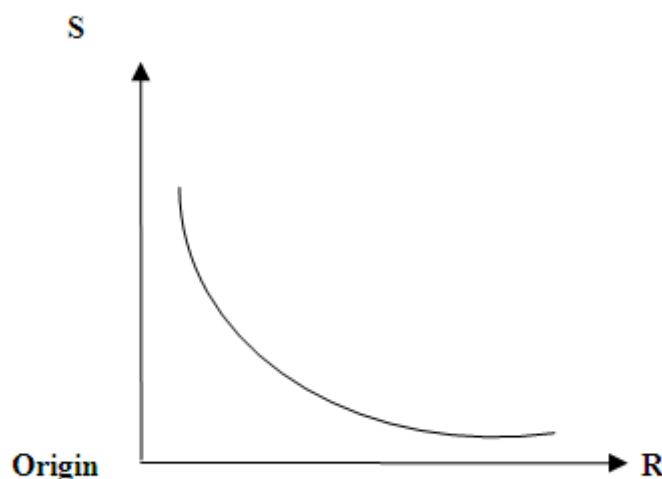


Figure: 8

III. OPTIMIZATION TO SEARCH THE NODES IN SOCIAL NETWORK

In mathematics, computer science, economics etc. **optimization** (alternatively, **optimization** or **mathematical programming**) is the selection of a best element (with regard to some criteria) from some set of available alternatives. In the simplest case, an optimization problem consists of maximizing or minimizing a real function by systematically choosing input values from within an allowed set and computing the value of the function. The generalization of optimization theory and techniques to other formulations comprises a large area of applied mathematics. More generally, optimization includes finding "best available" values of some objective function given a defined domain (or a set of constraints), including a variety of different types of objective functions and different types of domains. In the social network, there is also exist optimization method to search the required objects (nodes).

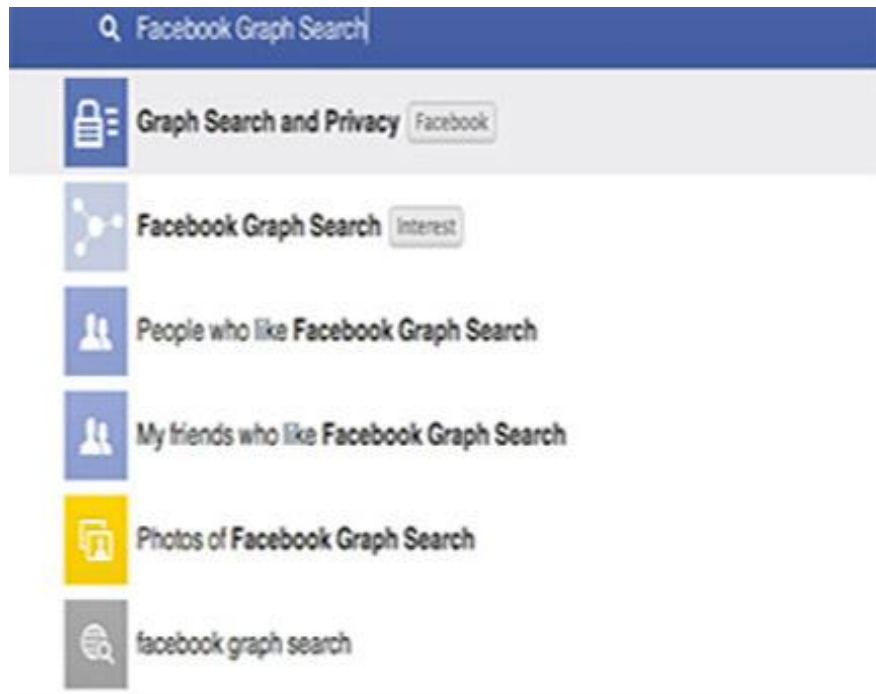


Figure: 9



Figure: 10

IV. SOCIAL NETWORKS AND FINITE STATE AUTOMATION

A social networks may be considered as a very large finite state automation with much more complex structure which normally we do not have in any small Finite state machine.

Social networks may be considered as a Finite State Machine (FSM) : $(Q, \Sigma, \delta, q_0, F)$

Where Q =All Non Empty nodes

Σ = All Non-Null symbol or all messages which are not NULL

q_0 = Initial node =any member of the entire social networks

F =Final node

δ =Transition Function which $Q \times \Sigma$ to some node in Q

This idea may help the user to reduce a big social network to a reduced states network. Similarly in general a social network will be a Non deterministic finite state automation. One can apply the successor table method or epsilon closure method to convert NDFSA to DFA. This may give quite interesting results in social networks. The authors will do this exercise in much more elaborate way in the next work.

A Simple example show this:

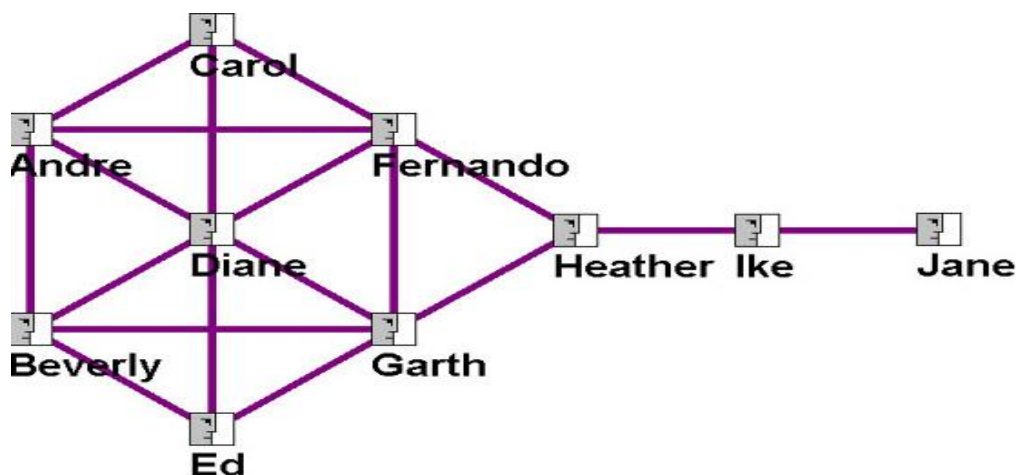


Figure:11

V. CONCLUSION AND FUTURE SCOPE

In the present paper the authors mainly tried to explore the relationship among the various persons in a particular social network. The authors tried to find the answer to the question that what is reliability and what is security in social area networks. The reliability and security of data is a very important issue in any social networks. The authors also pointed out that optimization methods may be applied to optimize the social networks paths. The authors also tried to find the relationship with finite state automation with a social networks. In the future work the authors will extensively use the concept of FSM in social area networks. There is open scope to apply Finite State Automation in social area networks.

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