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Modeling Heavy Tails in BSE Sensex

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***Abstract:** Recent trends in stock market shows that large deviation that cannot be modeled using normal distribution. Large deviation means that models using normal distribution are unable to account the risk in the market. Stable distribution is highly flexible distribution that can model heavy tails and skewness. A stable distribution is defined by four parameters (α,β,γ,δ) α index of stability, β skewness parameter, γ scale parameter δ location parameter γ and δ are equivalent to standard deviation and mean in normal distribution. Generalized Central Limit Theorem shows that if the finite variance assumption is dropped, the only possible resulting limits are stable distribution*

***Keywords:** Stock market, Heavy tailed data, Stable distribution, generalized central limit theorem*

I. INTRODUCTION

High frequency trading uses computer algorithms to execute large number of orders at very fast speed time. As the name suggest the trading of securities happen at a very rapid pace that that is at the speed of mille seconds this reduction of time to execute the trade is achieved by fully automate computer programs that analyze the data and make split second decisions. In this scenario the human's intervention is completely eliminated. The primary purpose is to increase liquidity in the market. But because all the algorithms use the same strategy at times they create temporary illiquidity that causes sharpe price changes in the market. The sudden and unexplained price changes in the market are known as Nora effect.

Normal Distribution is used to model the price fluctuations in the stock market but the changes in the stock shows much higher standard deviation (up to 14 standard deviations). 1987 market crash, LTCM collapse. 2007 market crash each and every one shows that higher deviation as predicted by normal distribution is quite normal. And since once in every decade the financial models fail miserably. May 6 2010 flash crash is yet another instance that sudden price changes are the Nome and not the exception.This leads to the conclusion that research community should go beyond normal distribution and IID random variables to explain Nora effect

Stable distributions are a general class of probability distributions that allow skewness and heavy tails and have many interesting mathematical properties. The class was found out by Paul Lévy in his study of sums of independent identically distributed terms in the 1920's. There is no closed formulas for densities and distribution functions for all but a few stable distributions (Gaussian, Cauchy and Lévy, see Figure 1), has been a major drawback to the use of stable distributions by practitioners. There are now reliable computer programs to compute stable distribution functions, densities and quartiles. With these computer programs, it is possible to use stable models in a variety of practical problems. The name stable is that they retain their shape both scale and shift under addition.

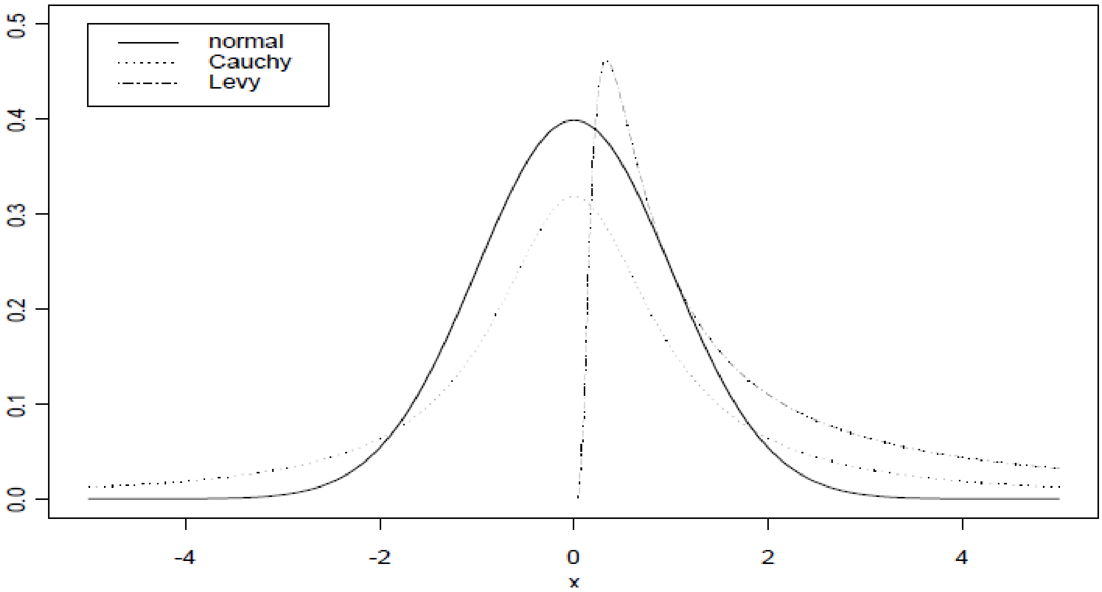


Figure 1: Graph of standardized Normal N(0,1), Cauchy(1,0) and Lévy(1,0) densities. Source : J P Nolan

Definition of stable Distribution

An important property of normal or Gaussian random variables is that the sum of two of them is itself a normal random variable. One of the result is that if X is normal, then for X1 and X2 independent copies of X and any positive constants a and b

$$aX1 + bX2 \overset{d}{=} cX + d \qquad (1)$$

For some positive c and some $d \in \mathbb{R}$. (The symbol $\overset{d}{=}$ means equality in distribution, i.e. both expressions have the same probability law.) In words, equation (1) says that the shape of X is preserved (up to scale and shift) under addition.

Definition 1: A random variable X is stable or stable in the broad sense if for X1 and X2 are independent copies of X and any positive constants a and b, (1) holds for some positive c and some $d \in \mathbb{R}$.The random variable is strictly stable or stable in the narrow sense if (1) holds with d = 0 for all choices of a and b. A random variable is symmetric stable if it is stable and symmetrically distributed around 0, e.g. X $\overset{d}{=} -$X.

Normal or Gaussian distributions $X \backsim N(\mu, \sigma^2)$ if it has a density

$$f(x) = \frac{1}{\sqrt{2\pi\sigma}}\ exp\left(-\frac{(x-\mu)^2}{2\sigma^2}\right), \qquad -\infty < x > \infty$$

The cumulative distribution function, for which there is no closed form expression, is

$F(x) = P(X \le x) = \phi\left(\frac{x-\mu}{\sigma}\right)$, Where $\phi(z)$ =probability that a standard normal random variable is less than or equal z.

Cauchy distributions $X \sim Cauchy(\gamma, \delta)$ If it has density

$$f(x) = \frac{1}{\pi}\frac{\gamma}{\gamma^2+(x-\delta)^2}, \quad -\infty < x > \infty$$

These are also called Lorentz distributions in physics. Cauchy distribution is stable with parameters a = 1, b = 0 and with given degrees of freedom of a Cauchy distribution.

Lévy distributions X$\sim Lévy(\gamma, \delta)$ if it has density

$$f(x) = \sqrt{\frac{\gamma}{2\pi}}\frac{1}{(x-\delta)^{\frac{3}{2}}}\ exp\left(-\frac{\gamma}{2(x-\delta)}\right), \quad \delta < x < \infty$$

L´evy distribution is stable with Parameters a = 1=2, b = 1.

C	P(X>c)		
	Normal	Cauchy	L´evy
0	0.5000	0.5000	1.0000
1	0.1587	0.2500	0.6827
2	0.0228	0.1476	0.5205
3	0:001347	0.1024	0.4363
4	0:00003167	0.0780	0.3829
5	0:0000002866	0.0628	0.3453

Definition 2: Non-degenerate X is stable if and only if for all n > 1, there exist constants

$c_n > 0$ and $d_n \in \mathbb{R}$ such that $X_1 + \cdots + X_n \overset{d}{=} c_n + d_n$, where $X_1, \ldots\ldots X_n$ are independent, identical copies of X. X is strictly stable if and only if $d_n = 0$ for all n.

The above definition means that scaling constant is described by the relationship $c_n = n^{1/\alpha}$

The most concrete way to describe all possible stable distributions is through the characteristic function or Fourier transform. (For a random variable X with distribution function F(x), the characteristic function is defined by $\phi(u) = Eexp(iuX) = \int_{-\infty}^{\infty} \exp(iux)\, dF(x)$. The function f (*u*) completely determines the distribution of *X* and has many useful mathematical properties, the sign function is used below, it is defined as

$$signu = \begin{cases} -1 & u < 0 \\ 0 & u = 0 \\ 1 & u > 0 \end{cases}$$

Definition 3: A random variable *X* is *stable* if and only if $X \overset{d}{=} aZ + b$, where $0 < \alpha \leq 2,\ -1 \leq \beta \leq 1,\ a > 0,\ b \in \mathbb{R}$ and *Z* is a random variable with characteristic function

$$Eexp(iuZ) = \begin{cases} \exp\left(-|u|^{\alpha}\left[1 - i\beta \tan\frac{\pi\alpha}{2}(signu)\right]\right) & \alpha \neq 1 \\ \exp\left(-|u|\left[1 - i\beta\frac{2}{\pi}(signu)\log|u|\right]\right) & \alpha = 1 \end{cases} \quad (2)$$

These distributions are symmetric around zero when b = 0 and *b* = 0, in which case the characteristic function of *aZ* has the simpler form

$$\phi(u) = e^{-a^{\alpha}|u|^{\alpha}}$$

A Normal (μ,σ^2) distribution is stable with (α =2, β =0, $a = \sigma/\sqrt{2}$, *b* = μ), a Cauchy(γ ;δ) distribution is stable with (α = 1, β = 0, *a* =γ, *b* =δ) and a $Lévy(\gamma, \delta)$ distribution is stable with (α = ½, β = 1*;a* =γ, *b* =δ).

There is no closed formulas to describe a stable distribution but with the new software that can accurately generate them. A general stable distribution requires four parameters to describe: An index of stability or characteristic exponent$0 < \alpha \leq 2$, a skewness parameter β between the range $-1 \leq \beta \leq 1$ If β =0 then the distribution is symmetric, if the β <0 then the distribution is skewed towards left and if β >0 then it is skewed towards right. A scale parameter γ can be any positive number $\gamma > 0$ and a location parameter $\delta \in \mathbb{R}$. We will use γ for the scale parameter and δ for the location parameter to avoid confusion with the symbols σ which is used for standard deviation and μ for mean. The parameters are restricted to the range a 2 (0; 2], b 2 [-1; 1], γ > 0 and δ $\delta \in \mathbb{R}$ R. Generally γ >0, although γ = 0 will sometimes be used to denote a degenerate distribution concentrated at d when it simplifies the statement of a result. Since γ & β determine the shape of the distribution, they may be considered shape parameters.

There are multiple parameterizations for stable laws and much confusion has been caused

Definition 4: A random variable X is S(α,β,γ,δ:0) if

$$X \overset{d}{=} \begin{cases} \gamma(Z - \beta \tan\frac{\pi\alpha}{2}) + \delta & \alpha \neq 1 \\ \gamma Z & \alpha = 1 \end{cases} \tag{3}$$

where $Z = Z$(a;b) is given by (1.2). X has characteristic function

$$Eexp(iux) = \begin{cases} \exp\left(-\gamma|u|^{\alpha}[1 + i\beta(\tan\frac{\pi\alpha}{2})(\mathrm{sign}u)(|\gamma u|^{1-\alpha} - 1)] + i\delta u\right) & \alpha \neq 1 \\ \exp\left(-\gamma|u|\left[1 + i\beta\frac{2}{\pi}\, signu\right)\log(\gamma|u|)\right] + i\delta u & \alpha = 1 \end{cases} \tag{4}$$

When the distribution is standardized, i.e. scale g = 1, and location d = 0, the symbol S(a,b;0) will be used as an abbreviation for S(a,b,1,0;0).

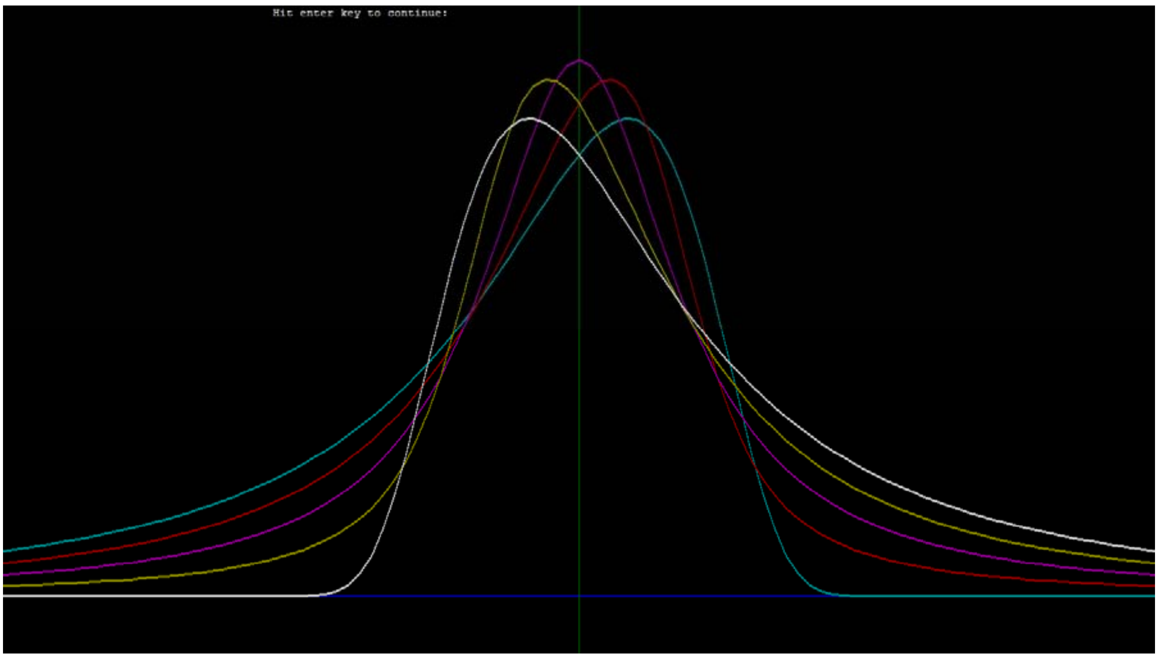
Definition 5: A random variable X is S (a,b,g,d;1) if

$$X \overset{d}{=} \begin{cases} \gamma z + \delta & \alpha \neq 1 \\ \gamma z + \left(\delta + \beta\frac{2}{\pi}\gamma log\gamma\right) & \alpha = 1 \end{cases} \tag{5}$$

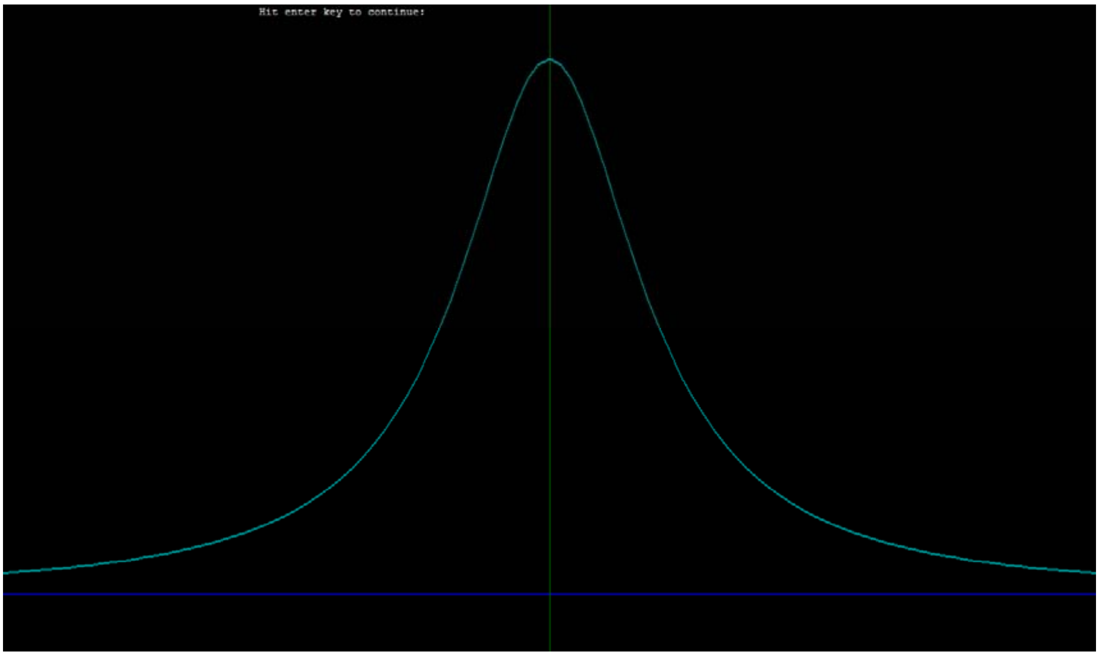
Where $Z = Z$(a;b) is given by (1.2). X has characteristic function

$$Eexp(iux) = \begin{cases} \exp\left(-\gamma^{\alpha}|u|^{\alpha}[1 + i\beta(\tan\frac{\pi\alpha}{2})(\mathrm{sign}u)] + i\delta u\right) & \alpha \neq 1 \\ \exp\left(-\gamma|u|\left[1 + i\beta\frac{2}{\pi}\, signu\right)\log(|u|)\right] + i\delta u) & \alpha = 1 \end{cases} \tag{6}$$

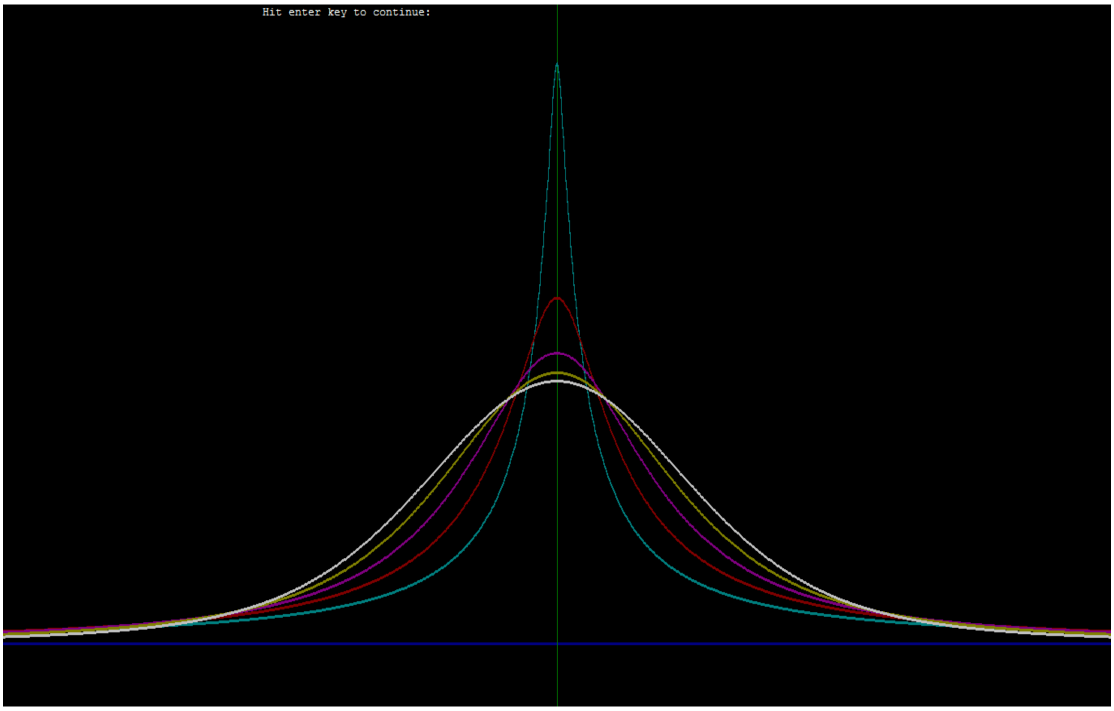
When the distribution is standardized, i.e. scale γ = 1, and location δ = 0, the symbol S(α,β;1) will be used as an abbreviation for S(,β,1,0;1).



Graph 2: β= -1, -0.5, 0 .5 and 1 – Source : Stable .exe



Graph 3: α= 2(Normal distribution) – Source: Stable.exe



Graph 4: α=0.5,0.75,1, 1.25 and 1.5 – Source Stable.exe

Generalized Central Limit Theorem

The classical Central Limit Theorem says that the normalized sum of independent, identical terms with a finite variance converges to a normal distribution. To be more precise, let $X1; X2; X3; : : :$ be independent identically distributed random variables with mean m and variance s 2. The classical Central Limit Theorem states that the sample mean $Xn = (X1+ \ldots.+Xn)/n$ will have

$$\frac{\bar{X}_n - \mu}{\sigma/\sqrt{n}} \xrightarrow{d} Z \sim N(1,0) \; as \; n \longrightarrow \infty$$

To match the notation in what follows, this can be rewritten as

$$a_n(X_1 + \cdots X_n) - b_n \overset{d}{\rightarrow} ZN(1,0)\ as\ n\ \rightarrow \infty$$

Where $a_n = 1/(\sigma\sqrt{n})\ and\ b_n = \sqrt{n}\mu/\sigma$

Theorem 6: The Generalized Central Limit Theorem shows that if the finite variance assumption is dropped, the only possible resulting limits are stable.

Generalized Central Limit Theorem A nondegenerate random variable Z is α-stable for some 0< α 2 if and only if there is an independent, identically distributed sequence of random variables $X_1, X_2, X_3, ..$ and constants $a_n > 0, b_n \in \mathbb{R}$ with

$$a_n(X_1\ ...\ X_{n)} - b_n \overset{d}{\rightarrow} Z$$

The following definition helps us to explain convergence of normalized sums.

Definition 7: A random variable *X* is in the *domain of attraction* of *Z* if and only if there exists constants $a_n > 0, b_n \in \mathbb{R}$ *with*

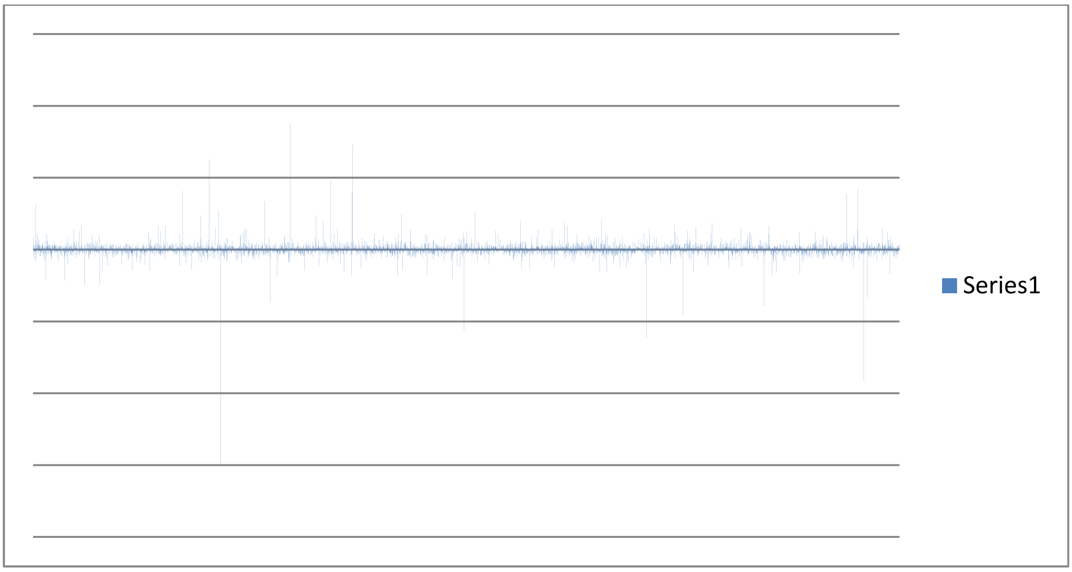
$$a_n(X_1\ ...\ X_{n)} - b_n \overset{d}{\rightarrow} Z$$

Where $X_1, X_2, X_3, ..$ are independent identically distributed copies of *X*. *DA*(*Z*) is the set of all random variables that are in the domain of attraction of *Z*.

Parameter of stable distribution

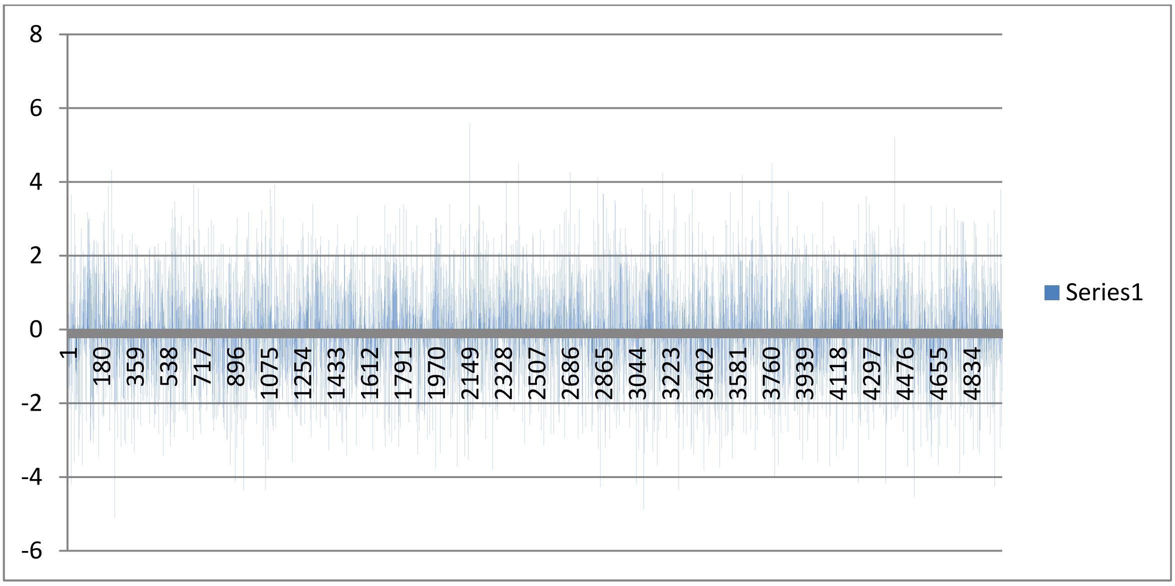
The after inputting the daily log returns of BSE sensex from 1991 to 2014 the parameters are estimated using Zolotarev (M) parameterization,

Alpha	Beta	Gama	Delta
1.5849	0.0675	0.1138	0.1291



Alpha	Beta	Gama	Delta
2	0	1	0

The above parameters are equivalent to normal distribution N (1,0)



II. Interpretation

The alpha value is 1.5849 which is clearly below that of normal distribution (value 2) but above that of Cauchy distribution (value 1). This means there is more deviation than normal distribution. This also means fat tails. Since alpha value is more than that of Cauchy distribution scale parameter (standard deviation) exists.

III. Conclusion

Stable distribution is n distribution that is more general form of distribution than other distribution. By adjusting the parameters we can generate all distribution. One distribution instead of many distributions, this is one step in the direction of unification of science. Since it has more symmetry and elegance it is an elegant way of representing the randomness of nature. Since it is elegant and beautiful it must be right. A good model should be flexible and stable distribution model can solve the problem of heavy tails that normal distribution failed to address. The outcome is that the risk model that uses stable distribution has higher deviation from average which means higher risk tolerance. In models that use normal distribution large deviations are swept under the carpet as outliners or anomalies but with stable distribution we can incorporate those observation in the model. Basel III norms has proposed higher capital requirements for banks, likewise the financial risk management companies that has exposure in different financial markets should hold more capital and reduce leverage.

IV. Suggestion

Firm trades in financial instruments uses traditional risk management tools which is based on normal distribution should migrate to stable distribution. Social science researchers should consider stable distribution instead of normal distribution.

Researchers should go from phenomenon to axiom not from axiom to phenomenon. What it mean is that we should create theories that fit the empherical observation and shoud not modify or fit empirical observation to theoty.

References

1. L. Bachelier - The Theory Of Speculation 1900 -Translated by D. May from Annales scientifiques de *l'*École Normale Supérieure, Sér. 3, 17 (1900), p. 21-86.
2. Eugene F. Fama – Behaviour of Stock Market Prices - The Journal of Business, Vol. 38, No. 1. (Jan., 1965), pp. 34-10
3. Burton G. Malkiel - The Eficient Market Hypothesis and Its Critics -Journal of Economic Perspectives—Volume 17, Number 1—Winter 2003—Pages 59 – 82
4. Enrico Scalas *and* Kyungsik Kim - The Art of Fitting Financial Time Series with Lévy Stable Distributions- Journal of the Korean Physical Society, Vol. 50, No. 1, January 2007, pp. 105»111
5. Ernst Eberlein and Wolfgang Kluge - Calibration of Lévy Term Structure Models

6. Andrew Matacz - Financial Modeling and Option Theory with the Truncated Levy Process - *Report 97-28, October 1997*

7. ´Alvaro Cartea and Sam Howison- Option Pricing with Lévy-Stable Processes Generated by Lévy -Stable Integrated Variance

8. Hélyette Geman- Pure Jump Lévy Processes for Asset Price Modelling- Journal of Banking and Finance, July 2002

9. Rafa l Weron- Levy-stable distributions revisited:- tail index > 2 does not exclude the Levy-stable regime- International Journal of Modern Physics C (2001) vol. 12 no. 2

10. Sebastian Raible- Lévy Processes in Finance:- Theory, Numerics, and Empirical Facts-PhD desitation

11. John P. Nolan- Stable Distributions - Models for Heavy Tailed Data

12. Mirasol A. Canedo and Edgardo D Cruz-The Philippine stock return and Levy Disributon

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