ISSN: 2321-7782 (Online)
e-ISJN: A4372-3114
Impact Factor: 6.047

Volume 5, Issue 8, August 2017

International Journal of Advance Research in Computer Science and Management Studies

Research Article / Survey Paper / Case Study

Available online at: www.ijarcsms.com

An Inventory Model for Deterioration Items with Imperfect Production and Price Sensitive Demand under Partial Backlogging

Gopal Pathak[1]
Dept. of Mathematics,
BKBIET, Pilani – India

Vipin Kumar[2]
Dept. of Mathematics,
BKBIET, Pilani – India

C. B. Gupta[3]
Dept. of Mathematics,
BITS, Pilani – India

Abstract: In this paper, an economic production quantity model for decaying items with deterministic demand has been developed. As the selling price is the main criterion of the consumer when they go to the market to buy a particular item therefore the deterministic demand rate is assumed as a function of selling price. In this model deterioration is considered and taken as time dependent. The imperfect production process is presented in this paper. The model is developed under shortages with partially backlogged with time dependent backlogging rate. Rate of production is taken as demand dependent. In order to find the optimal replenishment policies a mathematical model is presented. To illustrate the present study, numerical example and sensitivity analysis are also cited.

Keywords: Inventory, Deterioration, Imperfect Production, Shortages, Partial backlogging.

I. INTRODUCTION

In an inventory management, an important matter is the dealing of unfulfilled demands that arise during the shortages of the items in the existing stock. Most of the researcher has developed their model with the assumption that either shortage are completely backlogged or completely lost. But in real life situation it has been observed that during the period of shortage the rate of backlogging rate is smaller when the waiting time is longer. Therefore for the realistic business situation the rate of backlogging should be variable as well as dependent on the waiting time for the next replenishment. In literature it has been found that so many researchers have modified the inventory policies by considering the time proportional partial backlogging rate. Zangwill (1966) developed a production multi period production scheduling model with backlogging. Montgomery et al. (1973) presented inventory models with a mixture of backorders and lost sales. Rosenberg (1979) considered the analysis of a lot-size model with partial backlogging. Inventory models with partial backorders were proposed by Park (1982). Mak (1987) proposed optimal production-inventory control policies for an inventory system. Shortages in inventory were allowed and partially backlogged. Wee (1993) developed economic production lot size model for deteriorating items with partial backordering. Abad (1996) presented a generalized model of dynamic pricing and lot-sizing for perishable items. Shortage was allowed and the demand was partially backlogged. Wee (1999) developed deteriorating inventory model with quantity discount, pricing and partial backordering. Shortages in inventory were allowed and partially backlogged. Abad (2000-a) considered an optimal price and order size inventory policy for a reseller under partial backlogging. Abad (2000-b) studied the problem of determining the lot size for a perishable good under finite production. Shortage in inventory was allowed with partial backordering and lost sale. Papachristos and Skouri (2000) developed an EOQ inventory model over a finite planning horizon, with constant deterioration rate, time-dependent backlogging rate and time varying demand. The problem of determining the optimal price and lot size for a reseller was considered by Abad (2001). Selling price was taken as constant in the inventory cycle. Teng et al. (2002) presented an optimal replenishment policy for deteriorating items with time-varying demand and partial backlogging. Cost comparison of four inventory models for deteriorating items with variable demand was discussed by Skouri and Papachristos (2003-a). Shortage was allowed and assumed to be partially backlogged. An EOQ model with deteriorating items and partial backlogging was proposed by Skouri and Papachristos (2003-b).

Zhou et al. (2004) developed a general time-varying demand inventory lot-sizing model with waiting-time-dependent backlogging and a lot-size-dependent replenishment cost. Chu and Chung (2004) discussed the sensitivity of the inventory model with partial backorders. Ghosh and Chaudhuri (2005) considered an EOQ model with time varying deterioration and linear time varying demand over finite time horizon. Shortages in inventory were allowed and partially backlogged with waiting time dependent backlogging rate. An analytic solution of the model was discussed and it was illustrated with the help of a numerical example. Chang et al. (2006-a) then established an appropriate model in which building up inventory is profitable, and then provide an algorithm to find the optimal solution to the problem. A numerical was also used to illustrate the model numerically. An economic order quantity model for a retailer to determine its optimal selling price, replenishment number and replenishment schedule with partial backlogging was suggested by Chang et al. (2006-b). Dye et al. (2007-a) formulated deterministic inventory model for deteriorating items with price dependent demand. The demand and deterioration rates were continuous and differentiable function

of price and time, respectively. Shortages for unsatisfied demand were allowed and backlogging rate was taken as negative exponential function of the waiting time. Dye et al. (2007-b) presented a deterministic inventory model for deteriorating items with two warehouses. The deterioration rate in both the warehouses was taken as different. Shortages in OW were allowed and the backlogging demand rate was dependent on the duration of stock out. An inventory lot-size model for deteriorating items with partial backlogging was formulated by Chern et al. (2008). Authors have taken time value of money in to consideration. The demand was assumed to fluctuating function of time and the backlogging rate of unsatisfied demand was a decreasing function of the waiting time. The effects of inflation and time value of money were also considered in the model. Thangam and Uthayakumar (2008) presented a two-level supply chain model with partial backordering and approximated Poisson demand. Kumar Vipin, Pathak Gopal and Gupta C.B. (2013) developed a deterministic inventory model for deteriorating items with selling price dependent demand and parabolic time varying holding cost under trade credit.

In this paper, an economic production quantity model for decaying items with deterministic demand has been developed. As the selling price is the main criterion of the consumer when they go to the market to buy a particular item, therefore the deterministic demand rate is assumed as a function of selling price. In this model deterioration is considered and taken as time dependent. The imperfect production process is presented in this paper. The model is developed under shortages with partially backlogged with time dependent backlogging rate. Rate of production is taken as demand dependent. In order to find the optimal replenishment policies a mathematical model is presented. We have divided this paper in seven different sections. In the second, assumptions and notation are given for mathematical model formulations which are elaborated in the third section. Numerical illustration is mentioned in fourth sections and sensitivity analysis is mentioned in fifth sections of this paper. In the sixth section, observations are shown. In the seventh section, we have concluded our model.

II. Assumption and Notations

We consider the following assumptions:

(a) The production rate is dependent on demand rate.

(b) Deterioration is time varying.

(c) Demand is price dependent.

(d) Holing cost is time dependent.

(e) Shortages are allowed and partially backlogged.

We consider the following notations

$I(t)$: Inventory level at any time t.

P : Production rate $\{P = \alpha D(p)\}$.

α : Constant $(\alpha > 1)$.

D (p) : Demand rate $D(p) = \frac{\gamma}{p^{\delta}}$.

γ, δ : Constant Demand Parameter.

p : Selling price within the inventory cycle.

T : Duration of inventory cycle when there is positive inventory.

ψ : Duration of stock-out inventory.

$T + \lambda$: The time at which inventory level again becomes zero after satisfying backlogged demand.

λ : Constant.

β : The time up to which production occurs.

τ : Waiting time.

$\theta(t)$: Rate of Deterioration $\theta(t) = \theta t$.

θ : Constant.

ξ : Set up cost.

$B(\tau)$: Rate of backlogging during the stock-out period $B(\tau) = K_0 e^{-K_1 \tau}$.

K_0, K_1: Constants where $K_0 < 1$, $K_1 \geq 0$.

v : Unit cost of production.

$h_1 + h_2 t$: Inventory holding cost/ unit/ period.

Q : Total amount of production.

T.P. : Total profit.

T.A.P. : Total average profit.

III. Mathematical Model

The process starts with production at t = 0 and continue up to $t = \beta$. At this point the production stops and inventory depletes due to combined effect of demand and deterioration. At t = T the inventory level becomes zero and shortage occur.

$t = T + \psi$ is the time for max shortage. At this point production again starts and it satisfies backlogging. The differential equations governing the transition of the system are given by:

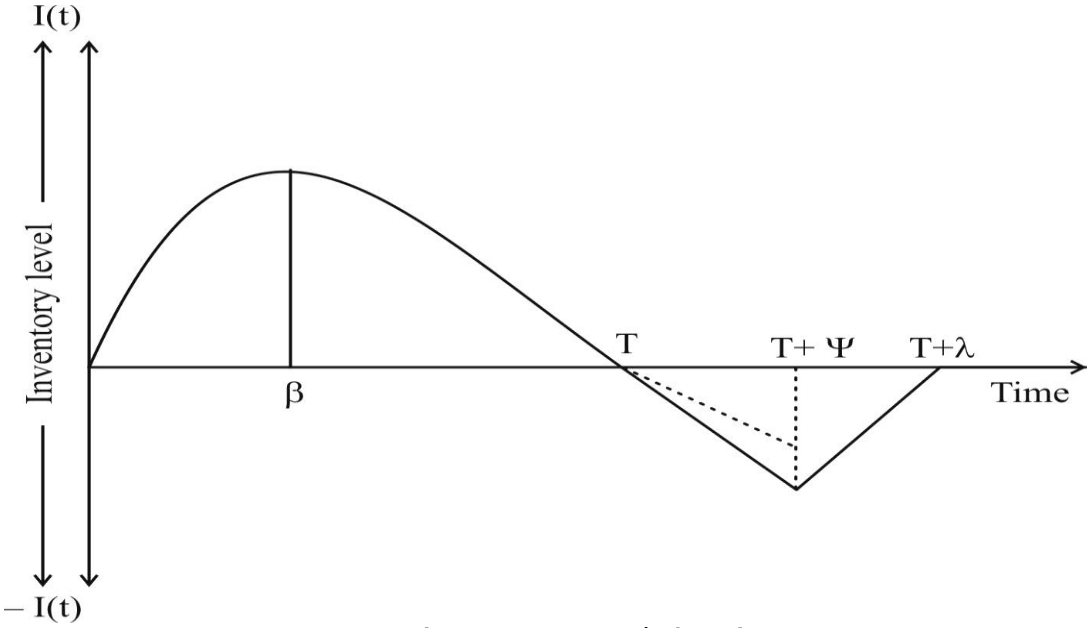

Fig. 1: Production inventory system for the vendor

$$\frac{dI(t)}{dt} = P - \theta t I(t) - D(p), \qquad 0 \le t \le \beta \qquad \ldots(1)$$

$$\frac{dI(t)}{dt} = -\theta t I(t) - D(p), \qquad \beta \le t \le T \qquad \ldots(2)$$

$$\frac{dI(t)}{dt} = -D(p)B(\tau), \qquad T \le t \le T + \psi \qquad \ldots(3)$$

$$\frac{dI(t)}{dt} = P - D(p)B(\tau), \qquad T + \Psi \le t \le T + \lambda \qquad \ldots(4)$$

with boundary conditions

$$I(0) = 0,\ I(T) = 0,\ I(T+\lambda) = 0 \qquad \ldots(5)$$

Solution of equation (1) is given by

$$I(t) = (\alpha - 1)\frac{\gamma}{p^{\delta}}\left(t + \frac{kt^3}{6}\right)e^{-kt^2/2}, \qquad 0 \le t \le \beta \qquad \ldots(6)$$

Solution of equation (2) is given by

$$I(t) = \frac{\gamma}{p^{\delta}}\left[(T - t) + \frac{k}{6}\left(T^3 - t^3\right)\right]e^{-kt^2/2}, \qquad \beta \le t \le T \qquad \ldots(7)$$

Using expressions (6) and (7) at $t = \beta$, we get

$$I(\beta) = \frac{\gamma}{p^{\delta}}\left[(T + \beta) + \frac{k}{6}\left(T^3 - \beta^3\right)\right]^{e^{-k\beta^2/2}} = (\alpha - 1)\frac{\gamma}{p^{\delta}}\left(\beta + \frac{k\beta^3}{6}\right)e^{-k\beta^2/2}$$

$$(T-\beta)+\frac{\theta}{6}\left(T^3-\beta^3\right)=(\alpha-1)\left(\beta+\frac{K\beta^3}{6}\right) \quad \ldots(8)$$

It is assumed that customers are impatient and at a given time $t \in [T, T+\psi]$ only a fraction of demand is backlogged. This fraction, denoted by $B(\tau)$, is a decreasing function of τ.

Specifically, we let

$$B(\tau)=K_0 e^{-k_1\tau} \qquad K_0<1, K_1\geq 0 \quad \ldots(9)$$

We assume that the customers are served on first come first served basis. At any given time $t\in[T, T+\lambda]$, τ would depend upon the backorders because all previous backorders are to be filled first before filling the new backorder.

As seen in figure, during the time span $(T, T+\psi)$ there is no production but only backlogging of demand occurs. Thus for $t\in[T, T+\psi]$ the waiting time is given by $\tau = T+\psi-t-\frac{I(t)}{P}$. In time span $[T+\psi, T+\lambda]$ the current backlog is depleted at a rate of R but at the same time new backorders are taken. Hence for $t\in[T+\psi, T+\lambda]$ the waiting time is given by $\tau=-I(t)/P$. Then we have:

$$\frac{dI}{dt}=-\frac{\gamma}{p^\delta}B(T+\psi-t-I(t)/P), \qquad T\leq t\leq t+\psi$$

Thus the solution of equation (3) is given by

$$I(t)=\frac{P}{K_1}\left(\log\left[\frac{P}{DK_0 e^{K_1(t-T-\psi)}+P-DK_0 e^{-K_1\psi}}\right]\right) \quad \ldots(10)$$

Similarly, for $t\in[T+\psi, T+\lambda]$,

$$\frac{dI}{dt}=P-DB\left(\frac{-I(t)}{P}\right), \qquad (T+\psi)\leq t\leq T+\lambda$$

$$\frac{dI}{dt}=P-DK_0 e^{-K_1\left(-I(t)/P\right)} \quad \ldots(11)$$

From equation (10), we have

$$I(T+\psi)=\frac{P}{K_1}\left(\log\left[\frac{P}{DK_0+R-DK_0 e^{-K_1\psi}}\right]\right) \quad \ldots(12)$$

Using boundary condition (12) the solution of equation (11) is given by

$$I(t)=\frac{P}{K_1}\left(\log\left[\frac{P}{P e^{-K_1(t-T-\psi)}-DK_0 e^{-K_1(t-T)}+DK_0}\right]\right), T+\psi\leq t\leq T+\lambda \quad \ldots(13)$$

We know that $I(T+\lambda)=0$, put this value in equation (13)

$$I(t+\lambda)=\frac{P}{K_1}\left(\log\left[\frac{P}{P\,e^{-K_1(\lambda-\psi)}-DK_0e^{-K_1\lambda}+DK_0}\right]\right)$$

$$0=\log\left[\frac{P}{P\,e^{-K_1(\lambda-\psi)}-DK_0e^{-K_1\lambda}+DK_0}\right]$$

$$P=P\,e^{-K_1\lambda}\,e^{K_1\psi}-DK_0e^{-K_1\lambda}+DK_0$$

$$\frac{P+DK_0e^{-K_1\lambda}-DK_0}{P\,e^{-K_1\lambda}}=e^{K_1\psi}$$

$$\psi=\frac{1}{K_1}\log\left[\frac{e^{K_1\lambda}(P-DK_0)+DK_0}{P}\right] \qquad \ldots(14)$$

Note that the production occurs in continuous time spans $[0,\beta]$ and $[T+\psi,T+\lambda]$. Hence the total production during this interval is

$$Q=P.\beta+P.(T+\lambda-T-\psi)$$

$$Q=P.\beta+P.(\lambda-\psi) \qquad \ldots(15)$$

Now Sales revenue = $p.DT+pP(\lambda-\psi)$ ...(16)

Production cost = $v\left(P\beta+P(\lambda-\psi)\right)$ ...(17)

Set up cost = ξ ...(18)

Holding cost = $\int_0^{\beta}(h_1+h_2t)I(t).dt+\int_{\beta}^{T}(h_1+h_2t)I(t).dt$

$$=\int_0^{\beta}(h_1+h_2t)(\alpha-1)\frac{\gamma}{p^{\delta}}\left(\frac{t+\theta t^3}{6}\right)e^{-\theta t^2/2}.dt$$

$$+\int_{\beta}^{T}(h_1+h_2t)\frac{\gamma}{p^{\delta}}\left[(T-t)+\frac{\theta}{6}\left(T^3-t^3\right)\right]e^{-\theta t^2/2}.dt$$

Holding cost $=(h_1+h_2\frac{\beta^2}{2})(\alpha-1)\frac{\gamma}{p^{\delta}}\left(\frac{\beta^2}{2}-\frac{\mu P\beta^4}{12}\right)$

$$+\frac{(h_1+h_2\frac{T^2}{2})\gamma}{p^{\delta}}\left\{\frac{T^2}{2}+\frac{\theta T^4}{12}-\left(T\beta-\frac{\beta^2}{2}\right)-\frac{\theta}{6}\left(T^3\beta-\frac{\beta^4}{4}\right)+\frac{\theta}{2}\left(\frac{T\beta^3}{3}-\frac{\beta^4}{4}\right)\right\} \ldots(19)$$

Deterioration cost = $\left(\alpha D\beta - DT\right)v$ …(20)

Now the total profit during the interval $\left[0, T+\lambda\right]$

T.P = Sales revenue – production cost – set up cost

– inventory holding cost – deterioration cost

$$T.P. = \left[p.\left(DT + \mu P\left(\lambda - \psi\right)\right) - v\left\{P\beta + P\left(\lambda - \psi\right)\right\} - \xi - (h_1 + h_2\frac{\beta^2}{2})(\alpha - 1)\frac{\gamma}{p^\delta}\left(\frac{\beta^2}{2} - \frac{\mu P\beta^4}{12}\right)\right.$$

$$-\frac{(h_1 + h_2\frac{T^2}{2})\gamma}{p^\delta}\left\{\frac{T^2}{2} + \frac{\theta T^4}{12} - \left(T\beta - \frac{\beta^2}{2}\right) - \frac{\theta}{6}\left(T^3\beta - \frac{\beta^4}{4}\right) + \frac{\theta}{2}\left(\frac{T\beta^3}{3} - \frac{\beta^4}{4}\right)\right\}$$

$$\left.-\frac{\theta}{6}\left(T^3\beta - \frac{\beta^4}{4}\right) + \frac{\theta}{2}\left(\frac{T\beta^3}{3} - \frac{\beta^4}{4}\right)\right\} - \left(\alpha D\beta - DT\right)v\right] \quad \text{…(21)}$$

$$T.A.P. = \frac{1}{T+\lambda}.T.P \quad \text{…(22)}$$

where D = $\frac{\gamma}{p^\delta}$

Now the problem is given by

Max. T.A.P

S.T. $T \geq 0, \lambda \geq 0$ and $p \geq v$ …(23)

IV. Numerical Example

In order to illustrate the preceding theory, we have taken a numerical example with the values of parameters in appropriate units as follows:

$$\delta = 2,\ D = 1{,}500{,}000p^{-3},\ P = \alpha D = 1850000p^{-3}, v = 12, \theta = 0.1, h_1 = 1.2,$$
$$h_2 = 0.5, K_0 = 0.9,\ K_1 = 0.6, p = 14, \gamma = 1{,}600{,}000,\ \lambda = 1.3, \xi = 400$$

Then, the optimal result for the system is

$p = 14.212,\ T = 1.44685,\ \lambda = 1.276,\ \beta = 0.1622,\ \psi = 0.755$ and the total profit of the EPQ model is 4587.94.

V. Sensitivity Analysis

Now we will check the sensitivity of various parameters as follows:

Table 1: Sensitivity Analysis in T with respect to θ

S. No.	θ	T	β	T.A.P.
1	0.04	3.36441	0.1150	4621.45

2	0.06	2.28024	0.1620	4602.68
3	0.08	1.44685	0.1652	4587.94
4	0.10	0.78577	0.2220	4576.05
5	0.12	0.24831	0.2561	4566.26

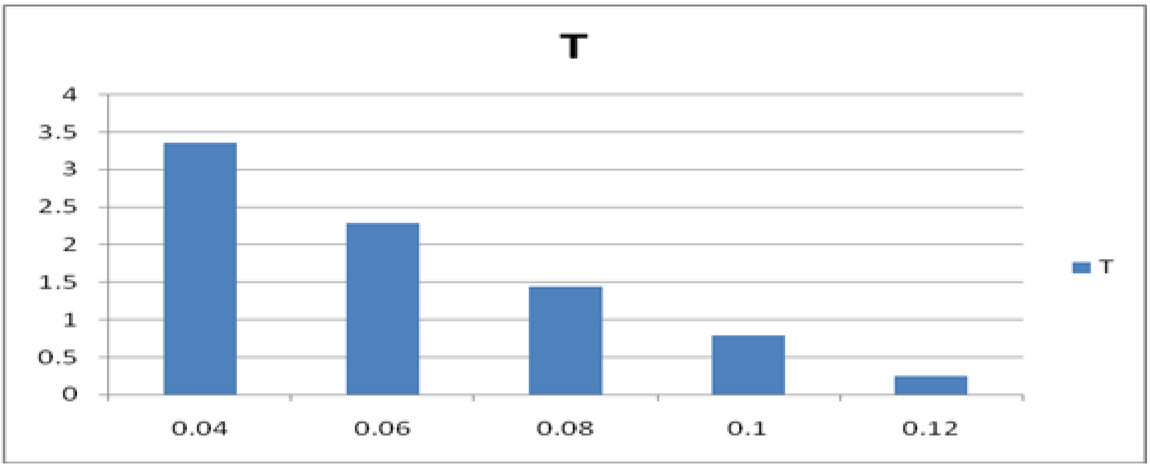

Fig. 2: Sensitivity of T w.r.t. ‘ θ ’

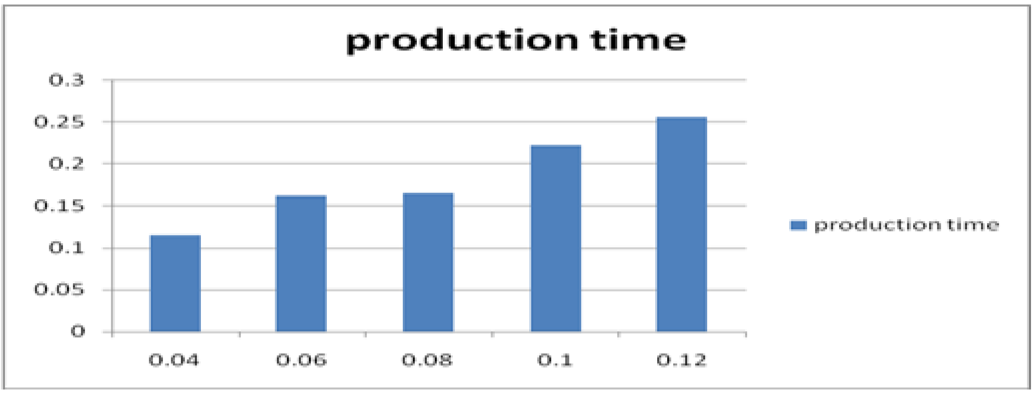

Fig. 3: Sensitivity of β w.r.t. θ

Table 2: Sensitivity Analysis in T and β for the variation in h_1

h_1	T	β
0.8	1.05200	0.6310
0.9	1.04312	0.6218
1.0	1.04300	0.6215
1.2	1.04500	0.6220
1.3	1.04510	0.6231

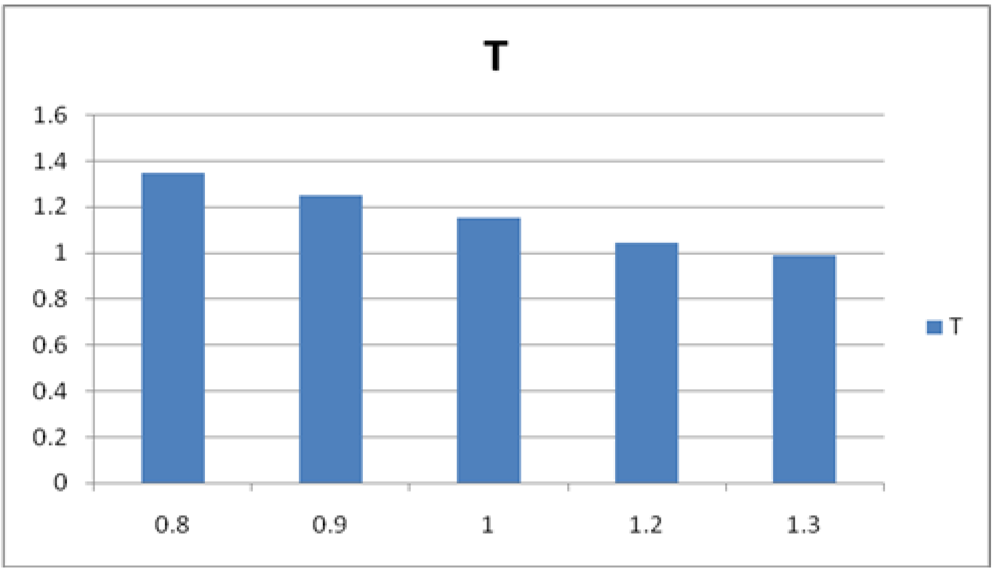

Fig. 4: Sensitivity of T w.r.t. h_1

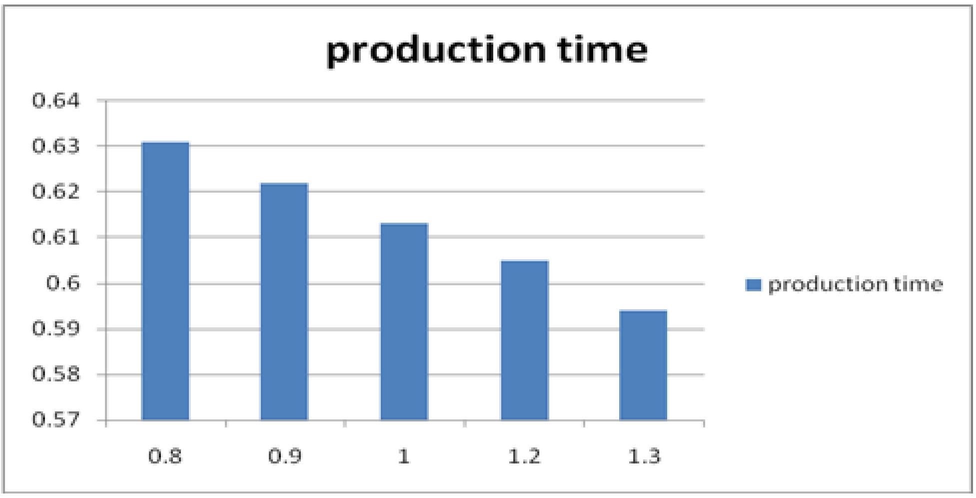

Fig. 5: Sensitivity of β w.r.t. h_1

VI. Observations

(i) From the table-1, it is clear that if θ increases then T decreases, β increases and total average profit decreases.

(ii) From the table-2, it is clear that if h_1 increases then T and β both first decreases for some values of h1 and then T and β both increases.

VII. Conclusion

In this chapter, we have developed an economic production inventory model for the imperfect production process. It deals with time varying deteriorating rate and selling price demand and partially backlogged shortages. It is well known that demand is affected by the price. Therefore, a flexible model of this type can be proved very beneficial for the policy makers. Such kind of a model ultimately reduces the stress upon the inventory manager to change his inventory keeping policy at every moment of time with the change of market position. In this way, this model helps to control the inventory without having to expend again and again to reformulate the inventory policy.

References

1. Abad, P.L. (1996), "Optimal pricing and lot-sizing under conditions of perishability and partial backlogging", Management Science, 42(8), 1093-1104.
2. Abad, P.L. (2000-a), "Optimal price and order size for a reseller under partial backordering", Computers & Operations Research (C.O.R.), 28(1), 53-65.
3. Abad, P.L. (2000-b), "Optimal lot size for a perishable good under conditions of finite production and partial backordering and lost sale", Computers & Industrial Engineering (C.I.E.), 38(4), 457-465.

4. Abad, P.L. (2001),"Optimal price and order size for a reseller under partial backordering", C.O.R.., 28, 53-65.
5. Chang C.T., Goyal S. K. and Teng J.T. (2006-a), "On an EOQ model for perishable items under stock-dependent selling rate and time-dependent partial backlogging", E.J.O.R., 174(2), 923-929.
6. Chang, H.J., Teng, J.T., Ouyang, L.Y. and Dye, C.Y. (2006-b), "Retailer's optimal pricing and lot-sizing policies for deteriorating items with partial backlogging", E.J.O.R., 168(1), 51-64.
7. Chern M.S., Yang H.L., Teng J.T. and Papachristos S.(2008),"Partial backlogging inventory lot-size models for deteriorating items with fluctuating demand under inflation", European Journal of Operational Research, 191,127-141.
8. Chu P. and Chung K.J. (2004), "The sensitivity of the inventory model with partial backorders", E.J.O.R., 152, 289-295.
9. Dye C.Y., Hsieh T.P. and Ouyang L.Y. (2007-a), "Determining optimal selling price and lot size with a varying rate of deterioration and exponential partial backlogging", E.J.O.R., 181(2), 668-678.
10. Dye C.Y., Ouyang L.Y. and Hsieh T.P. (2007-b), "Deterministic inventory model for deteriorating items with capacity constraint and time-proportional backlogging rate", E.J.O.R., 178 (3), 789-807.
11. Ghosh S.K. and Chaudhuri K.S. (2005), "An EOQ model for a deteriorating item with trended demand and variable backlogging with shortages in all cycles", A.M.O., 7(1), 57-68.
12. Kumar Vipin, Pathak Gopal and Gupta C.B.(2013),"A deterministic inventory model for deteriorating items with selling price dependent demand and parabolic time varying holding cost under trade credit", International journal of soft computing and Engineering(IJSCE),Vol-3, Issue-4,pp 33-36.
13. Mak K. L. (1987), "Determining optimal production-inventory control policies for an inventory system with partial backlogging", C.O.R., 14(4), 299-304.
14. Montogomery D.C., Bazaraa M.S. and Keshwani A.K. (1973), "Inventory models with a mixture of backorders and lost sales", Naval Research Logistics (N.R.L.), 20(2), 255-263.
15. Papachristosa S., Skouri K. (2000)," An optimal replenishment policy for deteriorating items with time-varying demand and partial–exponential type – backlogging", Operations Research Letters ,27 ,175–184.
16. Park, K.S. (1982), "Inventory models with partial backorders", I.J.S.S., 13, 1313-1317.
17. Rosenberg, D. (1979), "A new analysis of a lot size model with partial backlogging", N.R.L., 265, 349-353.
18. Skouri K., Papachristos S. (2003-a), "Four inventory models for deteriorating items with time varying demand and partial backlogging: A cost comparison", Optimal Control Applications and Methods, 24(6), 315 – 330.
19. Skouri K. and Papachristos S. (2003-b), "Optimal stopping and restarting production times for an EOQ model with deteriorating items and time dependent partial backlogging", I.J.P.E., 81-82, 525-531.
20. Teng J.T., Chang H.J., Dye C.Y. and Hung C.H. (2002), "An optimal replenishment policy for deteriorating items with time-varying demand and partial backlogging", O.R.L., 30, 387-393.
21. Thangam A. and Uthayakumar R. (2008), "A two-level supply chain with partial backordering and approximated Poisson demand", E.J.O.R., 187 (1), 228-242.
22. Wee H.M. (1993), "Economic production lot size model for deteriorating items with partial back ordering", C.I.E., 24(3), 449-458.
23. Wee, H.M. (1999), "Deteriorating inventory model with quantity discount, pricing and partial backordering", I.J.P.E., 59, 511-518.
24. Zangwill W.I. (1966), "A deterministic multi-period production scheduling model with backlogging", Management Science, 13, 105-119.
25. Zhou Y.W., Lau H.S. and Yang S.L. (2004), "A finite horizon lot-sizing problem with time-varying deterministic demand and waiting-time-dependent partial backlogging", I.J.P.E., 91(2), 109-119.