ISSN: 2321-7782 (Online)
Impact Factor: 6.047
Volume 4, Issue 8, August 2016
International Journal of Advance Research in Computer Science and Management Studies
Research Article / Survey Paper / Case Study
Available online at: www.ijarcsms.com

Convolution Structure of Quaternion Fractional Fourier-Mellin Transform

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***Abstract:** A quaternion is an extension of the complex number system. Quaternions have become a common part of mathematics and physics culture. It is specially useful in animation field and in filter designing. Quaternion convolution is useful in image processing.*

In this paper we introduced the definition of fractional quaternion Fourier-Mellin transform. Also we represented the new convolution structure of convolution with some basic properties such as linearity, shifting, associative, distributive and conjugation of quaternion convolution.

***Keywords:** Fourier-Mellin Transform, Fractional Fourier-Mellin Transform, Quaternion Fractional Fourier-Mellin Transform, Quaternion.*

I. INTRODUCTION

The Fourier-Mellin transform is an excellent tool in order to achieve translation, rotation and scale invariant shape recognition. Also has many applications such as registration of images, watermarks, invariant pattern recognition, preprocessing of images, registration of medical images, the comparison of plant leaves, reconstruction of the grayscale images, detecting watermark in images regardless of the scaling and rotation, detection of human face. S. Derrode, F. Ghorbel used the approximation of Fourier-Mellin transformation for the reconstruction of the grayscale images . Fourier-Mellin transform used the for detection of watermark in images regardless of the scaling and rotation. Using this transform human face also detected. G.S. Page used this method for comparing of distorted objects [8,9,10,11].

Quaternion is a backbone of animation field because quaternions express rotation as a rotation angle about a rotation axis. This is a more natural way to perceive rotation than Euler angle [3]. The 'vector' or 'pure quaternion' part was useful in physics, also used in electromagnetic theory , relativity and quantum mechanics [5].Convolution is a mathematical operation which has many applications in pure and applied mathematics such as numerical analysis, numerical linear algebra and the design and implementation of finite impulse response filters in signal processing [4]. Quaternion Fourier transforms is used to study hypoellipticity and to solve the heat equation [4].

Quaternion

A quaternion is a four-element vector that can be used to encode any rotation in a 3D coordinate system. Technically, a quaternion is composed of one real element and three complex elements, and it can be used for much more than rotations.

$i^2 = j^2 = k^2 = ijk = -1$

Here i, j, k represent 90^0 degree rotations about three mutually orthogonal axes. The other basic relationships:

$ij = k = -ji$

jk = i = -kj;

ki = j = -ik .

In our previous work we have proved the properties and convolution theorem for two dimensional fractional Fourier-Mellin transform [6,7]. In this work we have discussed the fractional Quaternion Fourier-Mellin transform, also the new convolution structure for fractional Quaternion Fourier-Mellin transform is presented. Moreover we have proved some basic properties of convolution.

II. Definitions

2.1 Definition of Two Dimensional Fractional Fourier-Mellin Transform

The two-dimensional fractional Mellin transform with parameters θ of $f(u,v)$ denoted by 2DFRMT{ $f(u,v)$} performs a linear operation, given by the integral transform. 2DFRMT

$$2DFRFMT\{ f(u,v)\} = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(x,y,u,v)\, K_{\alpha,\theta}(x,y,u,v,p,q,r,s)dxdydudv$$

where, $$K_{\alpha,\theta}(x,y,u,v,p,q,r,s) = \sqrt{\frac{1-icot\alpha}{2\pi}}\, e^{\frac{1}{2sin\alpha}[(x^2+y^2+p^2+q^2)cos\alpha-2(xp+yq)}$$

$$u^{\frac{2\pi ir}{sin\theta}-1} v^{\frac{2\pi is}{sin\theta}-1} e^{\frac{\pi i}{tan\theta}(r^2+s^2+log^2u+log^2v)}$$

2.2 Definition of Fractional Quaternion Fourier-Mellin Transform

For any two dimensional quaternion function $f(x,y,u,v)$ given by

$f(x,y,u,v) = f_r(x,y,u,v) + if_i(x,y,u,v) + jf_i(x,y,u,v) + kf_k(x,y,u,v)$ where $f_r(x,y,u,v), f_i(x,y,u,v), f_j(x,y,u,v), f_k(x,y,u,v)$ are real, the fractional quaternion Fourier-Mellin transform of $f(x,y,u,v)$is denoted by

$$FRFM^{i,j,k,l}_{\alpha_1,\alpha_2,\theta_1,\theta_2}(p,q,r,s) = FRFM^{i,j,k,l}_{\alpha_1,\alpha_2,\theta_1,\theta_2}\{f(x,y,u,v)\}$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} K^{i,k}_{\alpha_1,\theta_1}(x,u,p,r) f(x,y,u,v) K^{j,l}_{\alpha_2,\theta_2}(y,v,q,s)dxdydudv$$

where

$$K^{i,k}_{\alpha_1,\theta_1}(x,u,p,r) = \sqrt{\frac{1-icot\emptyset_1}{2\pi}}\, u^{\frac{2\pi ir}{sin\varphi_1}-1} e^{\frac{i}{2sin\emptyset_1}[(x^2+p^2)cos\emptyset_1-2xp]+\frac{\pi i}{tan\varphi_1}(r^2+log^2u)},$$

$$\emptyset_1 = \alpha_1\frac{\pi}{2}\ ,\ \varphi_1 = \theta_1\frac{\pi}{2}$$

$$K^{j,l}_{\alpha_2,\theta_2}(y,v,q,s) = \sqrt{\frac{1-icot\emptyset_2}{2\pi}}\, v^{\frac{2\pi is}{sin\varphi_2}-1} e^{\frac{i}{2sin\emptyset_2}[(y^2+q^2)cos\emptyset_2-2yq]+\frac{\pi i}{tan\varphi_2}(s^2+log^2v)},$$

$$\emptyset_2 = \alpha_2\frac{\pi}{2}\ ,\varphi_2 = \theta_2\frac{\pi}{2}\ .$$

III. Convolution Structure for Quaternion Fractional Fourier-Mellin Transform

Theorem: For any real, scalar or complex signal $f(x,y,u,v)$ and convolution kernel $g(x,y,u,v)$ and

$$h(x,y,u,v) = (f * g)(x,y,u,v)$$

$$= \sqrt{\frac{1-icot\emptyset_1}{2\pi}}\sqrt{\frac{1-icot\emptyset_2}{2\pi}}\, e^{\frac{-i}{2}(x^2cot\emptyset_1+y^2cot\emptyset_2)-\pi i[\frac{log^2u}{tan\varphi_1}+\frac{log^2v}{tan\varphi_2}}\{e^{\frac{i}{2}[(x^2cot\emptyset_1+y^2cot\emptyset_2)+\pi i[\frac{log^2u}{tan\varphi_1}+\frac{log^2v}{tan\varphi_2}}[f(x,y,u,v) * g(x,y,u,v)]$$
$$e^{\frac{i}{2}[(x^2cot\emptyset_1+y^2cot\emptyset_2)+\pi i[\frac{log^2u}{tan\varphi_1}+\frac{log^2v}{tan\varphi_2}}\}$$

where, * is the Quaternion Fractional Fourier-Mellin Transform convolution operator then

$$F_{\alpha_1,\alpha_{2,\theta_{1,\theta_2}}}\{h(x,y,u,v)\}(p,q,r,s) = e^{\frac{i}{2}[(p^2cot\emptyset_1+q^2cot\emptyset_2)-\pi i[\frac{r^2}{tan\varphi_1}+\frac{s^2}{tan\varphi_2}]}$$

$$FRFM^{i,j,k,l}_{\alpha_1,\alpha_{2,\theta_{1,\theta_2}}}\{f(\xi,\eta,\lambda,\mu)\}FRFM^{i,j,k,l}_{\alpha_1,\alpha_{2,\theta_{1,\theta_2}}}\{g(z,n,q,s)\}$$

Proof-

$$F_{\alpha_1,\alpha_{2,\theta_{1,\theta_2}}}\{h(x,y,u,v)\}(p,q,r,s)$$

$$= \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(x,y,u,v)\, K_{\alpha,\theta}(x,y,u,v,p,q,r,s)dxdydudv$$

$$= \sqrt{\frac{1-icot\emptyset_1}{2\pi}}\sqrt{\frac{1-icot\emptyset_2}{2\pi}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(x,y,u,v)\, e^{\frac{i}{2}(x^2cot\emptyset_1+p^2cot\emptyset_1+y^2cot\emptyset_2+q^2cot\emptyset_2)}$$

$$e^{-i(xpcosec\emptyset_1+yqcosec\emptyset_2)}u^{\frac{2\pi ir}{sin\varphi_1}-1}v^{\frac{2\pi is}{sin\varphi_2}-1}e^{\frac{\pi i}{tan\varphi_1}[r^2+log^2u]}\, e^{\frac{\pi i}{tan\varphi_2}[s^2+log^2v]}dxdydudv$$

$$= \sqrt{\frac{1-icot\emptyset_1}{2\pi}}\sqrt{\frac{1-icot\emptyset_2}{2\pi}}\, e^{\frac{i}{2}[(p^2cot\emptyset_1+q^2cot\emptyset_2)+\pi i[\frac{r^2}{tan\varphi_1}+\frac{s^2}{tan\varphi_2}]}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(x,y,u,v)\, e^{\frac{i}{2}(x^2cot\emptyset_1+y^2cot\emptyset_2)}$$

$$e^{-i(xpcosec\emptyset_1+yqcosec\emptyset_2)}u^{\frac{2\pi ir}{sin\varphi_1}-1}v^{\frac{2\pi is}{sin\varphi_2}-1}e^{\frac{\pi i}{tan\varphi_1}log^2u}\, e^{\frac{\pi i}{tan\varphi_2}log^2v}dxdydudv$$

From given

$$= \left(\sqrt{\frac{1-icot\emptyset_1}{2\pi}}\right)^2\left(\sqrt{\frac{1-icot\emptyset_2}{2\pi}}\right)^2 e^{\frac{i}{2}[(p^2cot\emptyset_1+q^2cot\emptyset_2)+\pi i[\frac{r^2}{tan\varphi_1}+\frac{s^2}{tan\varphi_2}]}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} e^{\frac{i}{2}(x^2cot\emptyset_1+y^2cot\emptyset_2)}$$
$$e^{-i(xpcosec\emptyset_1+yqcosec\emptyset_2)}$$

$$u^{\frac{2\pi ir}{sin\varphi_1}-1}v^{\frac{2\pi is}{sin\varphi_2}-1}\{\, e^{\frac{\pi i}{tan\varphi_1}log^2u}\, e^{\frac{\pi i}{tan\varphi_2}log^2v}\, e^{\frac{-i}{2}[(x^2cot\emptyset_1+y^2cot\emptyset_2)-\pi i(\frac{log^2u}{tan\varphi_1}+\frac{log^2v}{tan\varphi_2})}\,[e^{\frac{i}{2}[(x^2cot\emptyset_1+y^2cot\emptyset_2)+\pi i(\frac{log^2u}{tan\varphi_1}+\frac{log^2v}{tan\varphi_2})}$$

$$f(x,y,u,v) * g(x,y,u,v)\quad e^{\frac{i}{2}[(x^2cot\emptyset_1+y^2cot\emptyset_2)+\pi i(\frac{log^2u}{tan\varphi_1}+\frac{log^2v}{tan\varphi_2})}]\}dxdydudv$$

$$= \left(\sqrt{\frac{1-icot\emptyset_1}{2\pi}}\sqrt{\frac{1-icot\emptyset_2}{2\pi}}\right)^2 e^{\frac{i}{2}[(p^2cot\emptyset_1+q^2cot\emptyset_2)+\pi i[\frac{r^2}{tan\varphi_1}+\frac{s^2}{tan\varphi_2}]}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} e^{-i(xpcosec\emptyset_1+yqcosec\emptyset_2)}$$

$$u^{\frac{2\pi ir}{sin\varphi_1}-1}v^{\frac{2\pi is}{sin\varphi_2}-1}dxdydudv\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}\frac{1}{cf}\tilde{f}\,(a,b,c,f)\tilde{g}\,\left(x-a,y-b,\frac{u}{c},\frac{v}{f}\right)dadbdcdf$$

Putting $x-a=t,\ y-b=z,\quad \frac{u}{c}=m, \frac{v}{f}=n,$

$$x=t+a,\ y=z+b,\quad \frac{u}{m}=c, \frac{v}{n}=f$$

Differentiate with respect to t, z, m, n respectively

$$da = dt,\ db = dz,\ \frac{-u}{m^2} = dc,\ \frac{-v}{n^2} = df$$

$$= \left(\sqrt{\frac{1-icot\emptyset_1}{2\pi}}\sqrt{\frac{1-icot\emptyset_2}{2\pi}}\right)^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_0^{\infty}\int_0^{\infty} e^{\frac{i}{2}[(p^2cot\emptyset_1+q^2cot\emptyset_2)}\ e^{\pi i[\frac{r^2}{tan\varphi_1}+\frac{s^2}{tan\varphi_2}]}\ e^{-i(xpcosec\emptyset_1+yqcosec\emptyset_2)}$$

$$u^{\frac{2\pi ir}{sin\varphi_1}-1} v^{\frac{2\pi is}{sin\varphi_2}-1} dxdydudv\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_0^{\infty}\int_0^{\infty}\frac{1}{mn}\tilde{f}\left(x-t, y-z, \frac{u}{m}, \frac{v}{n}\right)\tilde{g}\ (t,z,m,n)\ dtdzdmdn\}$$

$$= \left(\sqrt{\frac{1-icot\emptyset_1}{2\pi}}\sqrt{\frac{1-icot\emptyset_2}{2\pi}}\right)^2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_0^{\infty}\int_0^{\infty} e^{\frac{i}{2}(p^2cot\emptyset_1+q^2cot\emptyset_2)}\ e^{\pi i[\frac{r^2}{tan\varphi_1}+\frac{s^2}{tan\varphi_2}]}\ e^{-i(xpcosec\emptyset_1+yqcosec\emptyset_2)}$$

$$u^{\frac{2\pi ir}{sin\varphi_1}-1} v^{\frac{2\pi is}{sin\varphi_2}-1} dxdydudv\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_0^{\infty}\int_0^{\infty}\frac{1}{mn}\tilde{f}\left(x-t, y-z, \frac{u}{m}, \frac{v}{n}\right)\tilde{g}\ (t,z,m,n)\ e^{\frac{i}{2}[(x-t)^2cot\emptyset_1+(y-z)^2cot\emptyset_2]}$$

$$e^{\pi i[\frac{log^2\frac{u}{m}}{tan\varphi_1}+\frac{log^2\frac{v}{n}}{tan\varphi_2}]}\ g(t,z,m,n)e^{\frac{i}{2}(t^2cot\emptyset_1+z^2cot\emptyset_2)}\ e^{\pi i(\frac{log^2 m}{tan\varphi_1}+\frac{log^2 n}{tan\varphi_2})} dtdzdmdn\}$$

Putting $x - t = \xi\ ,\ \ y - z = \eta,\ \frac{u}{m} = \lambda,\ \frac{v}{n} = \mu$

$$dx = d\xi,\quad dy = d\eta\ ,\quad du = md\lambda,\quad dv = nd\mu$$

$$x = \xi + t,\quad y = \eta + z,\quad u = \lambda m,\quad v = \mu n$$

$$= e^{\frac{i}{2}[(p^2cot\emptyset_1+q^2cot\emptyset_2)-\pi i[\frac{r^2}{tan\varphi_1}+\frac{s^2}{tan\varphi_2}]}\ \{\sqrt{\frac{1-icot\emptyset_1}{2\pi}}\sqrt{\frac{1-icot\emptyset_2}{2\pi}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_0^{\infty}\int_0^{\infty} f(\xi,\eta,\lambda,\mu)\ e^{\frac{i}{2}(\xi^2cot\emptyset_1+\eta^2cot\emptyset_2+p^2cot\emptyset_1+q^2cot\emptyset_2)}$$

$$e^{-i[\xi pcosec\emptyset_1+\eta qcosec\emptyset_2]}\ (\lambda)^{\frac{2\pi ir}{sin\varphi_1}-1}(\mu)^{\frac{2\pi is}{sin\varphi_2}-1} e^{\pi i(\frac{r^2}{tan\varphi_1}+\frac{s^2}{tan\varphi_2}+\frac{log^2\lambda}{tan\varphi_1}+\frac{log^2\mu}{tan\varphi_2})}\ d\xi d\eta d\lambda d\mu\}$$

$$\{\sqrt{\frac{1-icot\emptyset_1}{2\pi}}\sqrt{\frac{1-icot\emptyset_2}{2\pi}}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_0^{\infty}\int_0^{\infty} g(t,z,m,n)\ e^{\frac{i}{2}(t^2cot\emptyset_1+z^2cot\emptyset_2+p^2cot\emptyset_1+q^2cot\emptyset_2)} e^{-i[tpcosec\emptyset_1+zqcosec\emptyset_2]}$$

$$(m)^{\frac{2\pi ir}{sin\varphi_1}-1}(n)^{\frac{2\pi is}{sin\varphi_2}-1} e^{\pi i(\frac{r^2}{tan\varphi_1}+\frac{s^2}{tan\varphi_2}+\frac{log^2 m}{tan\varphi_1}+\frac{log^2 n}{tan\varphi_2})}\ dtdzdmdn\}$$

$$= e^{\frac{i}{2}[(p^2cot\emptyset_1+q^2cot\emptyset_2)-\pi i[\frac{r^2}{tan\varphi_1}+\frac{s^2}{tan\varphi_2}]}$$

$$\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_0^{\infty}\int_0^{\infty}\sqrt{\frac{1-icot\emptyset_1}{2\pi}}\ e^{\frac{i}{2}(\xi^2+p^2)cot\emptyset_1-i\xi pcosec\emptyset_1}(\lambda)^{\frac{2\pi ir}{sin\varphi_1}-1} f(\xi,\eta,\lambda,\mu)\ e^{\pi i\left(\frac{r^2}{tan\varphi_1}+\frac{log^2\lambda}{tan\varphi_1}\right)}\}$$

$$\sqrt{\frac{1-icot\emptyset_2}{2\pi}}\ e^{\frac{i}{2}(\eta^2+q^2)cot\emptyset_2-i\eta qcosec\emptyset_2}\ (\mu)^{\frac{2\pi is}{sin\varphi_2}-1} e^{\pi i(\frac{s^2}{tan\varphi_2}+\frac{log^2\mu}{tan\varphi_2})}\ d\xi d\eta d\lambda d\mu\}$$

$$\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_0^{\infty}\int_0^{\infty}\sqrt{\frac{1-icot\emptyset_1}{2\pi}}\ g(t,z,m,n)\ e^{\frac{i}{2}(t^2+p^2)cot\emptyset_1-itpcosec\emptyset_1}(m)^{\frac{2\pi ir}{sin\varphi_1}-1} e^{\pi i(\frac{r^2}{tan\varphi_1}+\frac{log^2 m}{tan\varphi_1})}\ g(t,z,m,n)$$

$$\sqrt{\frac{1-icot\emptyset_2}{2\pi}}\ e^{\frac{i}{2}(z^2+q^2)cot\emptyset_2-izqcosec\emptyset_2}\ (n)^{\frac{2\pi is}{sin\varphi_2}-1} e^{\pi i(\frac{s^2}{tan\varphi_2}+\frac{log^2 n}{tan\varphi_2})}\ dtdzdmdn\} = e^{\frac{i}{2}[(p^2cot\emptyset_1+q^2cot\emptyset_2)-\pi i[\frac{r^2}{tan\varphi_1}+\frac{s^2}{tan\varphi_2}]}$$ ------------ -(3.1)

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_0^{\infty}\int_0^{\infty}\{K_{\alpha_1,\theta_1}^{i,k}\ (\xi,\lambda,p,r)f(\xi,\eta,\lambda,\mu)K_{\alpha_2,\theta_2}^{j,l}(\eta,\mu,q,s)\}d\xi d\eta d\lambda d\mu$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_0^{\infty}\int_0^{\infty}\{K_{\alpha_1,\theta_1}^{i,k}\ (t,m,p,r)g(t,z,m,n)K_{\alpha_2,\theta_2}^{j,l}(z,n,q,s)\}dtdzdmdn$$

$$= e^{\frac{i}{2}[(p^2 cot\emptyset_1+q^2 cot\emptyset_2)-\pi i[\frac{r^2}{tan\varphi_1}+\frac{s^2}{tan\varphi_2}]} \ FRFM^{i,j,k,l}_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}\{f(\xi,\eta,\lambda,\mu)\}FRFM^{i,j,k,l}_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}\{g(z,n,q,s)\}\text{-------------(3.2)}$$

Hence proved.

IV. Some Basic Performance of Quaternion Convolution

4.1 Linearity property

Prove that

(i) $(A_1 f + A_2 g) * h = A_1(f * h) + A_2(g * h)$

(ii) $h * (A_1 f + A_2 g) = A_1(h * f) + A_2(h * g)$where $A_1, A_2 \epsilon H$

Proof-

(i)Consider

$LHS = (A_1 f + A_2 g) * h$

$= \beta * h \qquad (\because \beta = (A_1 f + A_2 g))$

By using (3.1) and (3.2) we get

$LHS = \ \beta * h$

$$= e^{\frac{-i}{2}[(p^2 cot\emptyset_1+q^2 cot\emptyset_2)-\pi i[\frac{r^2}{tan\varphi_1}+\frac{s^2}{tan\varphi_2}]} \ FRFM^{i,j,k,l}_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}\{\beta(x,y,u,v)\}FRFM^{i,j,k,l}_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}\{h(m,n,t,z)\}$$

$$= e^{-\frac{i}{2}(p^2 cot\emptyset_1+q^2 cot\emptyset_2)-\pi i[\frac{r^2}{tan\varphi_1}+\frac{s^2}{tan\varphi_2}]} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_0^{\infty}\int_0^{\infty} \beta(x,y,u,v)\, K_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}(x,y,u,v,p,q,r,s)dxdydudv$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_0^{\infty}\int_0^{\infty} h(m,n,t,z)\, K_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}(m,n,t,z,p,q,r,s)dmdndtdz$$

$$= e^{-\frac{i}{2}(p^2 cot\emptyset_1+q^2 cot\emptyset_2)-\pi i[\frac{r^2}{tan\varphi_1}+\frac{s^2}{tan\varphi_2}]}$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_0^{\infty}\int_0^{\infty}\{[A_1 f(x,y,u,v) + A_2 g(x,y,u,v)]K_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}(x,y,u,v,p,q,r,s)dxdydudv\}$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_0^{\infty}\int_0^{\infty} h(m,n,t,z)\, K_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}(m,n,,t,z,p,q,r,s)dmdndtdz\text{-----------------(4.1)}$$

Now consider

$RHS = A_1(f * h) + A_2(g * h)$

By using equation (3.1)and (3.2)

$$= e^{-\frac{i}{2}(p^2 cot\emptyset_1+q^2 cot\emptyset_2)-\pi i[\frac{r^2}{tan\varphi_1}+\frac{s^2}{tan\varphi_2}]} \ A_1\{FRFM^{i,j,k,l}_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}(f * h)\} + A_2\{FRFM^{i,j,k,l}_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}(g * h)\}$$

$$= e^{-\frac{i}{2}(p^2 cot\emptyset_1+q^2 cot\emptyset_2)-\pi i[\frac{r^2}{tan\varphi_1}+\frac{s^2}{tan\varphi_2}]}\{A_1 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_0^{\infty}\int_0^{\infty} f(x,y,u,v)\, K_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}(x,y,u,v,p,q,r,s)dxdydudv$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_0^{\infty}\int_0^{\infty} h(m,n,t,z)\, K_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}(m,n,,t,z,p,q,r,s)dmdndtdz\}$$

$$+\{A_2 \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_0^{\infty}\int_0^{\infty} g(x,y,u,v)\, K_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}(x,y,u,v,p,q,r,s)dxdydudv$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_0^{\infty}\int_0^{\infty} h(m,n,t,z)\, K_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}(m,n,,t,z,p,q,r,s)dmdndtdz\}$$

$$= e^{-\frac{i}{2}(p^2 cot\emptyset_1+q^2 cot\emptyset_2)-\pi i[\frac{r^2}{tan\varphi_1}+\frac{s^2}{tan\varphi_2}]} \quad \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_0^{\infty}\int_0^{\infty} h(m,n,t,z)\, K_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}(m,n,,t,z,p,q,r,s)dmdndtdz$$

$\{A_1 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_0^{\infty} \int_0^{\infty} f(x,y,u,v)\; K_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}(x,y,u,v,p,q,r,s)dxdydudv$

$+\{A_2 \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_0^{\infty} \int_0^{\infty} g(x,y,u,v)\; K_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}(x,y,u,v,p,q,r,s)dxdydudv$

$= e^{-\frac{i}{2}(p^2 cot\emptyset_1 + q^2 cot\emptyset_2) - \pi i[\frac{r^2}{tan\varphi_1} + \frac{s^2}{tan\varphi_2}]} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_0^{\infty} \int_0^{\infty} h(m,n,t,z)\, K_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}(m,n,,t,z,p,q,r,s)dmdndtdz$

$\{\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_0^{\infty} \int_0^{\infty} [A_1 f(x,y,u,v) + A_2 g(x,y,u,v)]\;\; K_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}(x,y,u,v,p,q,r,s)dxdydudv$ -------------------(4.2)

From (4.1) and(4.2) result (i)

$(A_1 f + A_2 g) * h = A_1(f * h) + A_2(g * h)$ is proved.

Similarly we can prove resut (ii)

$h * (A_1 f + A_2 g) = A_1(h * f) + A_2(h * g)$

4.2 Shifting property

Prove that (i) $(\alpha f * g) = \alpha(f * g)$

(ii) $(f * \alpha g) = \alpha(f * g)$

Proof- Consider,

LHS= $(\alpha f * g)$

By using equation (3.1) and (3.2)

$= e^{-\frac{i}{2}(p^2 cot\emptyset_1 + q^2 cot\emptyset_2) - \pi i[\frac{r^2}{tan\varphi_1} + \frac{s^2}{tan\varphi_2}]}\;\; FRFM^{i,j,k,l}_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}\{\alpha f(x,y,u,v)\} FRFM^{i,j,k,l}_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}\{g(m,n,t,z)\}$

$= e^{-\frac{i}{2}(p^2 cot\emptyset_1 + q^2 cot\emptyset_2) - \pi i[\frac{r^2}{tan\varphi_1} + \frac{s^2}{tan\varphi_2}]} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_0^{\infty} \int_0^{\infty} \{\alpha f(x,y,u,v)\} K_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}(x,y,u,v,p,q,r,s)dxdydudv$

$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_0^{\infty} \int_0^{\infty} \{g(m,n,t,z)\}\, K_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}(m,n,t,z,p,q,r,s)dmdndtdz$

$= e^{-\frac{i}{2}(p^2 cot\emptyset_1 + q^2 cot\emptyset_2) - \pi i[\frac{r^2}{tan\varphi_1} + \frac{s^2}{tan\varphi_2}]} \alpha \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_0^{\infty} \int_0^{\infty} \{f(x,y,u,v)\} K_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}(x,y,u,v,p,q,r,s)dxdydudv$

$\int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \int_0^{\infty} \int_0^{\infty} \{g(m,n,t,z)\}\, K_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}(m,n,t,z,p,q,r,s)dmdndtdz$

$= e^{-\frac{i}{2}(p^2 cot\emptyset_1 + q^2 cot\emptyset_2) - \pi i[\frac{r^2}{tan\varphi_1} + \frac{s^2}{tan\varphi_2}]}\;\; \alpha\{FRFM^{i,j,k,l}_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}[f(x,y,u,v)]\}\{FRFM^{i,j,k,l}_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}[g(m,n,t,z)]\}$

$= \alpha(f * g)$ Hence proved.

Similarly we can proved result

$(f * \alpha g) = \alpha(f * g)$

4.3 Distributive Property

Prove that $f * (g + h) = (f * g) + (f * h)$

Proof-

Consider,

$LHS = \; f * (g + h)$

$= f * \varphi \qquad (\because \varphi = g + h)$

By using equation (3.1) and (3.2) we get

$LHS = f * \varphi$

$$= e^{\frac{-i}{2}[(p^2 cot\emptyset_1 + q^2 cot\emptyset_2) - \pi i[\frac{r^2}{tan\varphi_1} + \frac{s^2}{tan\varphi_2}]} FRFM^{i,j,k,l}_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}\{f(x,y,u,v)\} FRFM^{i,j,k,l}_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}\{\varphi(m,n,t,z)\}$$

$$= e^{-\frac{i}{2}(p^2 cot\emptyset_1 + q^2 cot\emptyset_2) - \pi i[\frac{r^2}{tan\varphi_1} + \frac{s^2}{tan\varphi_2}]} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(x,y,u,v)\, K_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}(x,y,u,v,p,q,r,s) dxdydudv$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} \varphi(m,n,t,z)\, K_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}(m,n,t,z,p,q,r,s) dmdndtdz$$

$$= e^{-\frac{i}{2}(p^2 cot\emptyset_1 + q^2 cot\emptyset_2) - \pi i[\frac{r^2}{tan\varphi_1} + \frac{s^2}{tan\varphi_2}]}$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(x,y,u,v)\, K_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}(x,y,u,v,p,q,r,s) dxdydudv$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} (g+h)(m,n,t,z)\, K_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}(m,n,t,z,p,q,r,s) dmdndtdz$$

$$= e^{-\frac{i}{2}(p^2 cot\emptyset_1 + q^2 cot\emptyset_2) - \pi i[\frac{r^2}{tan\varphi_1} + \frac{s^2}{tan\varphi_2}]} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(x,y,u,v)\, K_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}(x,y,u,v,p,q,r,s) dxdydudv$$

$$\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} g(m,n,t,z)\, K_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}(m,n,t,z,p,q,r,s) dmdndtdz$$

$$+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} h(m,n,t,z)\, K_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}(m,n,t,z,p,q,r,s) dmdndtdz\}$$

$$= e^{-\frac{i}{2}(p^2 cot\emptyset_1 + q^2 cot\emptyset_2) - \pi i[\frac{r^2}{tan\varphi_1} + \frac{s^2}{tan\varphi_2}]} \{ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(x,y,u,v)\, K_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}(x,y,u,v,p,q,r,s) dxdydudv$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} g(m,n,t,z)\, K_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}(m,n,t,z,p,q,r,s) dmdndtdz$$

$$+ \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} f(x,y,u,v)\, K_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}(x,y,u,v,p,q,r,s) dxdydudv$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} h(m,n,t,z)\, K_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}(m,n,t,z,p,q,r,s) dmdndtdz\}$$

$= (f * g) + (f * h)$

4.4 Associative Property

Prove that-

$(f * g) * h = f * (g * h)$

Proof-

Consider

$LHS = (f * g) * h$

$= \delta * h \qquad (\because \delta = f * g)$

By using equation (3.1) and (3.2) we get

$$= e^{-\frac{i}{2}(p^2 cot\emptyset_1 + q^2 cot\emptyset_2) - \pi i[\frac{r^2}{tan\varphi_1} + \frac{s^2}{tan\varphi_2}]} FRFM^{i,j,k,l}_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}\{\delta(x,y,u,v)\}\ FRFM^{i,j,k,l}_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}\{h(m,n,t,z)\}$$

$$= e^{-\frac{i}{2}(p^2 cot\emptyset_1 + q^2 cot\emptyset_2) - \pi i[\frac{r^2}{tan\varphi_1} + \frac{s^2}{tan\varphi_2}]} \int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty} \{\delta(x,y,u,v)\}\quad K_{\theta_1,\theta_2}(x,y,u,v,p,q,r,s) dxdydudv$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}\{h(m,n,t,z)\}K_{\theta_1,\theta_2}(m,n,t,z,p,q,r,s)dmdndtdz$$

$$= e^{-\frac{i}{2}(p^2cot\emptyset_1+q^2cot\emptyset_2)-\pi i[\frac{r^2}{tan\varphi_1}+\frac{s^2}{tan\varphi_2}]}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}\{(f*g)(x,y,u,v)\}K_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}(x,y,u,v,p,q,r,s,)dxdydudv$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}\{\{h(m,n,t,z)\}K_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}(m,n,t,z,p,q,r,s)dmdndtdz$$

$$= e^{-\frac{i}{2}(p^2cot\emptyset_1+q^2cot\emptyset_2)-\pi i[\frac{r^2}{tan\varphi_1}+\frac{s^2}{tan\varphi_2}]}\quad\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}[e^{-\frac{i}{2}(p^2cot\emptyset_1+q^2cot\emptyset_2)-\pi i[\frac{r^2}{tan\varphi_1}+\frac{s^2}{tan\varphi_2}]}[FRFM^{i,j,k,l}_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}f(x,y,u,v)$$

$$FRFM^{i,j,k,l}_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}g(x,y,t,z)]K_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}(x,y,u,v,p,q,r,s)dxdydudv]$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}e^{-\frac{i}{2}(p^2cot\emptyset_1+q^2cot\emptyset_2)-\pi i[\frac{r^2}{tan\varphi_1}+\frac{s^2}{tan\varphi_2}]}\{h(m,n,t,z)\}K_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}(m,n,t,z,p,q,r,s)dmdndtdz$$

$$= e^{-\frac{i}{2}(p^2cot\emptyset_1+q^2cot\emptyset_2)-\pi i[\frac{r^2}{tan\varphi_1}+\frac{s^2}{tan\varphi_2}]}\;e^{-\frac{i}{2}(p^2cot\emptyset_1+q^2cot\emptyset_2)-\pi i[\frac{r^2}{tan\varphi_1}+\frac{s^2}{tan\varphi_2}]}$$

$$\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}f(x,y,u,v)\,K_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}(x,y,u,v,p,q,r,s)dxdydudv$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}g(m,n,t,z)\,K_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}(m,n,t,z,p,q,r,s)dmdndtdz]\,K_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}(x,y,u,v,p,q,r,s)dxdydudv\}$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}h(m,n,t,z)\}K_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}(m,n,t,z,p,q,r,s)dmdndtdz$$

$$= e^{-\frac{i}{2}(p^2cot\emptyset_1+q^2cot\emptyset_2)-\pi i[\frac{r^2}{tan\varphi_1}+\frac{s^2}{tan\varphi_2}]}$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}e^{-\frac{i}{2}(p^2cot\emptyset_1+q^2cot\emptyset_2)-\pi i[\frac{r^2}{tan\varphi_1}+\frac{s^2}{tan\varphi_2}]}f(x,y,u,v)\,K_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}(x,y,u,v,p,q,r,s)dxdydudv]$$

$$\{\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}\quad[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}g(m,n,t,z)\,K_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}(m,n,t,z,p,q,r,s)dmdndtdz$$

$$[\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}h(m,n,t,z)\,K_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}(m,n,t,z,p,q,r,s)dmdndtdz]\quad K_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}(m,n,t,z,p,q,r,s)dmdndtdz\}$$

$$= f*(g*h)$$

4.5 Conjugation Property

Prove that-$\overline{(f*g)} = \bar{g}*\bar{f}$

Proof- By using equation (3.1) and (3.2) we get

$$f*g = e^{-\frac{i}{2}(p^2cot\emptyset_1+q^2cot\emptyset_2)-\pi i[\frac{r^2}{tan\varphi_1}+\frac{s^2}{tan\varphi_2}]}FRFM^{i,j,k,l}_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}\{f(x,y,u,v)\}FRFM^{i,j,k,l}_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}\{g(m,n,t,z)\}$$

$$= e^{-\frac{i}{2}(p^2cot\emptyset_1+q^2cot\emptyset_2)-\pi i[\frac{r^2}{tan\varphi_1}+\frac{s^2}{tan\varphi_2}]}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}\{f(x,y,u,v)\}K_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}(x,y,u,v,p,q,r,s)dxdydudv$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}\{g(m,n,t,z)\}K_{\alpha_1,\alpha_{2,\theta_1,\theta_2}}(m,n,t,z,p,q,r,s)dmdndtdz$$

$$\overline{f*g} = e^{\frac{i}{2}(p^2cot\emptyset_1+q^2cot\emptyset_2)+\pi i[\frac{r^2}{tan\varphi_1}+\frac{s^2}{tan\varphi_2}]}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}\{f(x,y,u,v)\}\sqrt{\frac{1+icot\emptyset_1}{2\pi}}\sqrt{\frac{1+icot\emptyset_2}{2\pi}}$$

$$u^{\frac{-2\pi ir}{sin\varphi_1}-1}e^{\frac{-i}{2sin\emptyset_1}[(x^2+p^2)cos\emptyset_1-2xp]-\frac{\pi i}{tan\varphi_1}(r^2+log^2u)}\;v^{\frac{-2\pi is}{sin\varphi_2}-1}e^{\frac{-i}{2sin\emptyset_2}[(y^2+q^2)cos\emptyset_2-2yq]-\frac{\pi i}{tan\varphi_2}(s^2+log^2v)}$$

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{0}^{\infty}\int_{0}^{\infty}\{g(m,n,t,z)\}\sqrt{\frac{1+icot\emptyset_1}{2\pi}}\sqrt{\frac{1+icot\emptyset_2}{2\pi}}\,t^{\frac{-2\pi ir}{sin\varphi_1}-1}e^{\frac{-i}{2sin\emptyset_1}[(m^2+p^2)cos\emptyset_1-2mp]-\frac{\pi i}{tan\varphi_1}(r^2+log^2t)}$$

$$z^{\frac{-2\pi is}{sin\varphi_2}-1} e^{\frac{-i}{2sin\emptyset_2}[(n^2+q^2)cos\emptyset_2-2nq]-\frac{\pi i}{tan\varphi_2}(s^2+log^2 z)}$$

$= \bar{g} * \bar{f}$, Hence prove.

Basic properties of Quaternion convolution

1	**Linearity Property**	(i) $(A_1 f + A_2 g) * h = A_1(f * h) + A_2(g * h)$ (ii) $h * (A_1 f + A_2 g) = A_1(h * f) + A_2(h * g)$
2	**Shifting Property**	(i) $(\alpha f * g) = \alpha(f * g)$ (ii) $(f * \alpha g) = \alpha(f * g)$
3	**Distributive Property**	$f * (g + h) = (f * g) + (f * h)$
4	**Associative Property**	$(f * g) * h = f * (g * h)$
5	**Conjugation Property**	$\overline{(f * g)} = \bar{g} * \bar{f}$

V. Conclusion

In this paper we have developed the fractional quaternion Fourier-Mellin transform. The new convolution structure for Fractional Quaternion Fourier-Mellin Transform is obtained which is useful in image processing. Some basic properties of quaternion convolution are proved.

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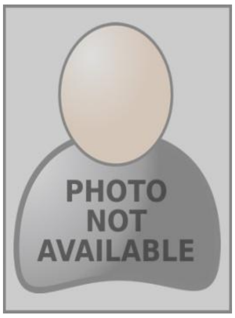

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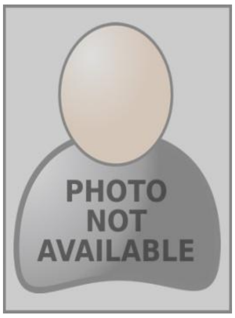

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