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A Theoretical Interpretation for the Study of Images Processing

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***Abstract:** In the present article and attempt has been made for the study of image processing, which is an important issue in various systems based on the electronic data analysis. This is also necessary for mankind in the age of modern technology, because it is essential to note systems of computer vision which render the indispensable help to the person in various situations and processes demanding special attention and also speed in decision-making. At the same time among variety of different methods and approaches for image processing it is necessary to allocate methodology of image normalization. The essence of such methodology of image processing consists in indemnification of different geometrical distortions of the input image which have been received a result of registration of the investigated image and its presentation on an input of system of the data analysis, in comparison with some reference picture. On the basis of such approach the work considers geometrical interpretation of the analyzed image of object as a basis of use of the device of the group theory.*

***Keywords:** Theoretical model, normalize, image processing, geometrical transformations and system of computer vision.*

I. INTRODUCTION

During the last decades, with the rapid growth of internet and digital technologies, computer vision systems have undergone quick advancement rendering the essential help to the person in different situations and processes demanding extraordinary attention and speed in decision-making. A standard of perception as some generalized picture of the reality are one of the sources of the information about outward things, processes which occur and also processes which could potentially occur, however cannot be accessible to a simple human sight. These circumstances impose certain features and restrictions, both on the nature of considered standards of perception, and on the possibilities of their analysis, additional data accessing about outward things.

Further, the standard of perception can be also formed in systems similar to human sight, such as the video shooting, a photo or could be transformed into images of visual perception of the human by means of any technical device – for example, it can be roentgenograms, tomography pictures and other pictures which are received with different special devices in the optical range or not. Hence, as a whole it is possible to speak about the images of visual perception having the different physical nature, but they can be transformed into images which are habitual to the human. It underlines the possibility of their generalization and consideration for the subsequent processing and analysis on the basis of approximately identical methods and approaches.

Among possible variety of images of visual perception the special spot is occupied with the images received in systems of technical or computer vision. Such systems are represented as some model of the human eye, capable to register an event in the real world and to react to these changes and without participation of men, on the basis of the transformation of the received information in images which are habitual for a human eye, and their subsequent analysis. Such attention regarding the systems of computer vision is caused by their ability to render the essential help in a human activity which is connected with automation of different technological processes in the industry or with processes which are remote for the direct visual control. At the same time, the work in an optical range of perception of pictures of the real world which are further exposed to some kind of the analysis is characteristic for the most of computer vision systems.

Example of such computer vision systems:

The computer vision system described in the work of R. Cucchiara, M. Piccardi and P. Mello. This system is intended for monitoring of the public transport traffic [1];

System of the video analysis of moving objects for research of sequence of varying scenes which is described in work of J. S. Jin, Z. Zhu and G Xu [2];

System of the visual analysis for a deterioration estimation of tools, offered in work of S. Kurada and C. Bradley [3];

System of recognition of objects and definition of their location for navigating needs is considered in work of A. Torralba, K. P. Murphy, W. T. Freeman and M. A. Rubin [4];

The automated system of computer vision for identification of plants by means of the automatic visual control [5].

A variety of methods of processing and analysis of the received images are put in a basis of the functioning of various systems of computer vision. In particular, there are methods of preliminary image processing (noise suppression, contrast increase, localization of separate sites of the image) [6], [7], methods of the preliminary analysis (segmentation, contour allocation) [8], [9], cognitive processing recognition methods of the received information [10], [11] and methods of the formalized representation of the received visual patterns for their subsequent processing [12].

Nevertheless, despite possibility of use of different methods of processing and analysis of the received visual patterns in various systems of computer vision it is necessary to proceed from both specificity of the representation of such images, and key problems the different systems of computer vision are used for. Thus, the normalization problem could be the main problem that should be solved in various systems of computer vision. The main point of normalization process consists in indemnification of the geometrical transformations received as a result of a deviation of the input image from the reference picture in comparison with a number of different base (reference) sets of possible images which have been investigated [13].

At the same time, there is both problem of initial formalization of the received visual patterns for their subsequent processing, and problem of formalization of normalization process. Basically the given problem is solved by each consideration of the functioning of one or another system of computer vision, because its solution is the formalized realization for corresponding system of computer vision.

For example, in work of S. Ganapathy, matrix transformation of an initial picture of a three-dimensional real object in two-dimensional image [12] is considered. However, at the same time S. Ganapathy does a conclusion about the complexity of the decision of an inverse problem [12] that it is possible to consider as one of the subtasks of normalization of images [13].

Separate questions of image normalization are considered in research of T. Sridevi, K. Swapna V. Kumar [14]. Nevertheless, T. Sridevi, K. Swapna V. Kumar don’t do the formalized generalizations, concerning interconnected consideration of representation questions and image normalization.

As a whole it is necessary to notice, that process of formalization of real pictures in the form of images for their subsequent analysis, processing and normalization is based on methods of classical geometry [15], [16]. Nevertheless, there are some open questions:

The formalized generalization of representation of images and their subsequent normalization for the purpose of unification of used methods for indemnification of geometrical transformations of images;

Achievement of comprehensible time frameworks of normalization process which is very labor intensive and expensive from the point of view of time of its realization [17].

Finally, it is caused by necessity of a finding of the comprehensible decisions on these questions within the limits of the given research. Differently, the main purpose of the given work is a consideration of formalization of models of representation and normalization of images from common positions where as a generalization it is offered to use the device of the group theory. Such choice is based on both natural calculation of geometrical transformations in the form of certain groups of transformations [18], and expediency of the single formalized description of representation of images and their normalization by means of the device of the group theory, that will be shown below.

II. An Overview for Investigated Objects and Their Images

As an object in our research we will consider the three-dimensional objects that don't change their absolute sizes in Euclid space. At image acquisition of such objects (on a film, on the screen) the two-dimensional picture of the object is formed and processed further.

Depending on an arrangement of the camera the object picture has one or other geometric distortion that should be compensated in the future. As a matter of fact, this is a general problem of normalization [13], [18]. Hence, this work investigates the problem of indemnification of geometrical distortions that allows to make the image more convenient view, both for recognition, and for storage and search.

For the solution of the given task we will consider the process of occurrence of possible geometrical distortions of the image and we will introduce a concept of the reference picture for such geometrical distortions will be considered.

As a reference picture of the image we will consider such picture of the image, on which the image is the most convenient (on a number of some predetermined properties and differences of considered object) for perception and recognition. As the object is three-dimensional, it will has some reference pictures (pictures from different perspectives of the object) according to them it is possible to receive other possible images of considered object.

Let's assume that the investigated object moves concerning the motionless camera (supervision points). Thus the object is concerning the camera on any distance, under any angle, with various displacements. Depending on this the received pictures will differ in their geometrical parameters.

Further, as the image we will consider a segmented picture "B" of investigating object O in some field of vision D. In this case as interpretation of the image for the purpose of its subsequent normalization we consider a comparison of the input image B with in advance set reference picture B_0.

The standard representation of the image (B or B_0) is the dimension matrix, $N \times M$ where N –quantity of columns, M – quantity of the lines.

Element $B(x,y)$ is a pixel of the image B with co-ordinates, (x,y), $1 \le x \le N$, $1 \le x \le M$ defines intensity (brightness) which can vary within K values of its level. If $K = 2$ then images are called binary [15], [16]. In this case $B(x,y)$ can possess only two values 0 or 1.

At $K > 2$ the images are called halftone, and the maximum value K quantifies gradation levels of brightness [15], [16]. In practice 256 values of brightness levels K are usually used. It defines so-called shade of gray in representation of the input image B for investigating object O [15], [18].

If the object image O is considered on a pure background, then it is considered, that at $(x, y) \in O$, $B(x, y)$ equals to value of brightness in this point, at $(x, y) \notin O$, $B(x, y) = 0$. In the most cases the systems of technical vision is enough for a work of image examination in shade of gray [18], however, the color analysis of the image is required sometimes. In this case the parameter K is compound and it is responsible for the color coding. In our given work we might intend, that considered input and reference pictures are presented in shade of gray.

III. Geometrical Interpretation of the Analyzed Image of Object on the Basic of Group theory

Let the observable object "O" changes its position in any way in the space. B_1 and B_2 are some two-dimensional pictures (images) of investigated object which are received in some other planes α and β (Figure 1).

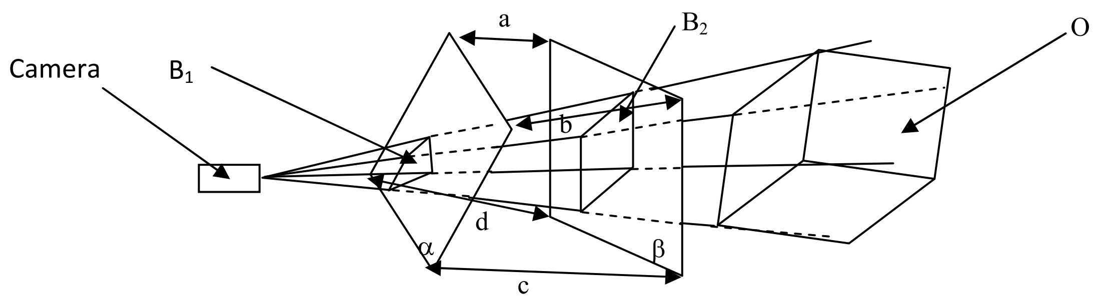

Figure 1. Scheme of occurrence of projective transformations

From classical geometry it is known [19], [20], that images B_1 and B_2 lying in the planes α and β between the distances a, b, c, d and has projective conformity, as shown in Figure 1. It is known, that various changes of the images of object can be described, by using projective group [18]. Thus, for the attainment of more universality of the mathematical model and its perception, also it is expedient to form it based on principles of the theory of projective transformations.

Further, it is known, that the projective transformation is unequivocal presented by the following matrix of the transformation which determinant is not equal to zero [18], [19], [20]:

$$\Pi = \begin{pmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{pmatrix},$$

where, $\det(\Pi) \neq 0$.

From every possible pictures of object let's arbitrarily choose some image B_0 which we will consider as the reference picture. In this case other images B (input images) of object can be received as a result of action of projective transformation on the reference picture. And vice versa, the input image can be rendered to the reference picture by influence of some projective transformation on it. In the latter case the following mathematical model describes communication of points of reference B_0 and input images B [18]:

$$B_0(x, y) = B\left(\frac{b_{11}x + b_{12}y + b_{13}}{b_{31}x + b_{32}y + b_{33}}, \frac{b_{21}x + b_{22}y + b_{23}}{b_{31}x + b_{32}y + b_{33}}\right) \quad (1)$$

where, the parameters b_{ij}, $i, j = 1,2,3$ parameters of projective transformation, and $\det(\Pi) = \det\|b_{ij}\| \neq 0$.

Representation on the bases of mathematical model and its perception the relation (1) is based on principles of the projective geometry was known earlier [18], [21], however its complexity consisting in multiparameterity and nonlinearity keeps an actual question about normalization of such transformations. Besides there is an invariancy requirement for the methods of image processing if the image have geometrical distortions. It means that results are identical irrespective of present transformation.

However it is necessary to consider a set of the projective transformations form group, which finally, gives possibility to apply the device of the group theory for working out of mathematical methods that are suitable for image processing. At the same time the concrete definition of separate subgroups of the projective group allows to define the ways of solution also raised the questions.

IV. Subgroups of Projective Group in Description of Possible Geometrical Distortions of the Analyzed Image

If $b_{31} = b_{32} = 0$ in the model of the image representation behind the formula (1) then from existence of the projective transformation follows that $b_{33} \neq 0$ (otherwise we will derive division by zero that contradicts to the existence of transformations).

Let's transform expression by (1) division by b_{33}, substitution of values $b_{31} = b_{32} = 0$ and making of following substitution:

per a_{11} denotes b_{11}/b_{33},

per a_{12} denotes b_{12}/b_{33},

and so on: $a_{13} = b_{13}/b_{33}$, $a_{21} = b_{21}/b_{33}$, $a_{22} = b_{22}/b_{33}$, $a_{23} = b_{23}/b_{33}$.

As a result we will derive mathematical model of communication between reference and input images which corresponds with affine model of perception [18], [19], [20]:

$$B_0(x, y) = B(a_{11}x + a_{12}y + a_{13}, a_{21}x + a_{22}y + a_{23}), \qquad (2)$$

where: $a_{11}, a_{12}, a_{13}, a_{21}, a_{22}, a_{23}$ – parameters of affine transformation.

The set of affine transformations forms a group (A).

Affine transformations are a special case of projective transformations and appear when $b_{31} = b_{32} = 0$, $b_{33} = 1$, it means that the plane of arrangement of observable object is parallel to the plane of camera. Then, the matrix of transformation which corresponds to this group has the following form:

$$A = \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ 0 & 0 & 1 \end{pmatrix}.$$

One of the major multiple parameter subgroups of affine group of transformation (A) is centroaffine group ($A_ц : a_{13} = a_{23} = 0$), images of which (input and reference) are connected by the following model's perception [18], [19], [20]:

$$B_0(x, y) = B(a_{11}x + a_{12}y, a_{21}x + a_{22}y). \qquad (3)$$

The process of transition from model's perception (2) to (3) corresponds to the procedure of a centering of images.

The displacements, compressions, turns and cross shift group are concerned to other basic subgroups of affine group. We will reduce the mathematical model's perception in each of these groups.

The mathematical model of displacement consists that the object image is displaced concerning co-ordinate axes in some field of vision without any changes of the geometrical sizes and its orientation [18], [19], [20]:

$$B_0(x,y) = B(x-n, y-m), \qquad (4)$$

where n, m – parameters of displacement along co-ordinate axes.

In Figure 2, the scheme of occurrence of transformation of mixture is presented, which forms group (C).

The group C in this mathematical model of perception represents the transformations of parallel transfers along co-ordinate axes; therefore we will designate it as $C_{xy}(n,m)$. And the displacement along one of co-ordinate axes (abscisses or ordinates) are designated accordingly $C_x(n)$ and $C_y(m)$.

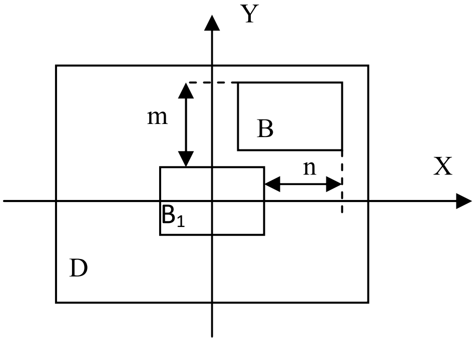

Figure 2. Scheme of occurrence of transformation of displacement

The matrixes corresponding to these groups of transformations, we will designate with the same letters. The given matrixes, following the general logic of representation of separate subgroups of the general group of projective transformations, have the following appearance:

$$C_{xy}(m,n)=\begin{pmatrix}1 & 0 & m\\ 0 & 1 & n\\ 0 & 0 & 1\end{pmatrix},\quad C_x(m)=\begin{pmatrix}1 & 0 & m\\ 0 & 1 & 0\\ 0 & 0 & 1\end{pmatrix},\quad C_y(n)=\begin{pmatrix}1 & 0 & 0\\ 0 & 1 & n\\ 0 & 0 & 1\end{pmatrix}.$$

Here in after group of transformations and the corresponding matrix we will designate equally.

Let the object " O " moves off from the camera (supervision points) in such a manner that all points of object move off on identical distance: ($A'A = B'B = C'C = D'D$) as shown is Figure 3. The derived images " B " of object " O " which are on the plane α will differ only in the scale factor.

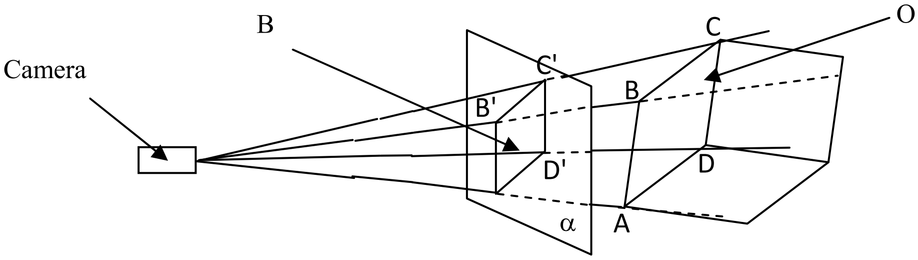

Figure 3. Scheme of occurrence of transformation of compression

In this case the mathematical model of vision represents transformation of uniform compression or «homothety» [22]) and looks like [18], [19], [20]:

$$B_0(x,y) = B(kx,ky), \tag{5}$$

where, k – compression parameter.

According to the mathematical model of non-uniform compression, it has two parameters such as:

$$B_0(x,y) = B(k_1x,k_2y), \tag{6}$$

where, k_1, k_2 – factors of compression along the co-ordinate axes.

Thus the matrix corresponds to the group of transformations of non-uniform compression appeared in the following form:

$$D_{xy}(k_1,k_2) = \begin{pmatrix} k_1 & 0 & 0 \\ 0 & k_2 & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Depending on values of parameters k_1, k_2 the transformation of compression of the image can be divided into following types [18]:

D – Homothety, at $k_1 = k_2$;

D_x – Compression along an abscissa at $k_2 = 1$;

D_y – Compression along a coordinate axis at $k_1 = 1$.

D_h – Hyperbolic turn at $k_1 = h$, $k_2 = 1/h$.

Let the object "O" does not move off from the camera (supervision points), i.e. all received images are considered in one plane β (Figure 4), and this object rotates round the axis which coincides with a direction of the camera axis. In this case the received images B_1 (image of a solid line) and B_2 (image of a dotted line) of the considered object "O" will be differed only by a turning angle θ. The turning angle θ is unique parameter in this model of vision. It is convenient to consider turning in two systems: Polar and Euclid. In polar system of co-ordinates the turning represents the displacement on the angle then in this case the mathematical model, is similarly represented to the usual parallel moving along the coordinate axis.

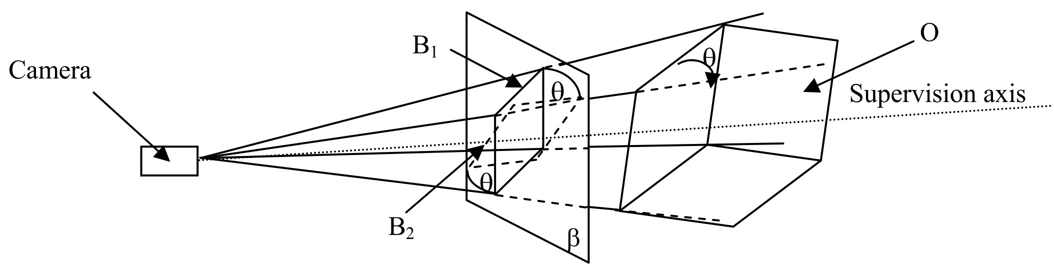

Figure 4. Scheme of occurrence of transformation of turning

If (ρ,φ) are the co-ordinates of the image's point in the polar system (ρ –distance, φ – angle), then the co-ordinate dependence on the reference and input images, $B(\rho, \varphi) = B_0(\rho, \varphi + \theta)$, where θ is displacement angle. In the Cartesian co-ordinates system the reference and input images are connected as other workers were found the same [18], [19], [20] in the following way:

$$B_0(x,y) = B(x\cos(\theta) + y\sin(\theta), -x\sin(\theta) + y\cos(\theta)), \quad (7)$$

where θ – turning angle.

Expression (7) is more often used as mathematical model of the description of transformations of turnings in connection with its simplicity at technical realization. As a whole the group of turnings is described by a matrix, the following general logic of representation of separate subgroups in the general group of projective transformations took place:

$$U(\theta) = \begin{pmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix}.$$

Figure 5 shows the example of transformation of cross shift: in which the rectangle B_0 is found after the transformation of cross shift with parameter $h = tg(\alpha)$.

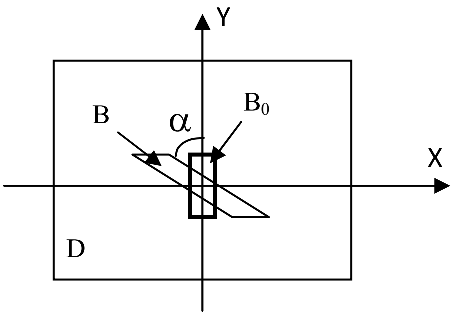

Figure 5. Scheme of occurrence of transformation of cross shift

Thus it is necessary to notice, that the transformation of cross shift happens along the coordinate axis, along the abscissa, and also along a straight line with some angular factor. However, in these cases the principle of occurrence of distortions is found similar.

The mathematical model of transformation along the abscissa looks like such as[18]:

$$B_0(x,y) = B(x + tg(\alpha)\cdot y, y), \quad (8)$$

where, α – angle of cross shift along the abscissa.

The group of transformations of the cross shift simultaneously along both axes is described by a matrix:

$$H_{xy} = \begin{pmatrix} 1 & tg(\alpha) & 0 \\ tg(\beta) & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix},$$

in which α and β set the angles of shift along the abscissa and ordinates accordingly.

Depending on the values of parameters, α and β group of cross shifts subdivided in [18]: H_x – cross shift along an abscissa at $tg(\beta) = 0$, $tg(\alpha) \neq 0$; and H_y – cross shift along a coordinate axis at $tg(\beta) \neq 0$, $tg(\alpha) = 0$.

The mathematical model of transformation along both the axes looks like [18]:

$$B_0(x,y) = B(x + tg(\alpha)y, tg(\beta)y), \quad (9)$$

where, α, β – transformation parameters.

The projective group of transformations has affine and also nonlinear transformations and many of them form a quantity of groups [18]. One of such groups forms a quantity of perspective transformations. The perspective transformations appear when the observable object is rotated around the axis which is perpendicular to the axis of supervision.

In this case the received images of the object will be differed by the parameter of perspective transformation to a certain angle of a turning [18], [19], [20], [23] and further they can pass in other quantity of images (the object is turned to the camera by other side). Change of object in planes is shown on Figure 6 (a) and Figure 6 (b), where B_1 and B_2 – examples of images of the object in planes α and β accordingly.

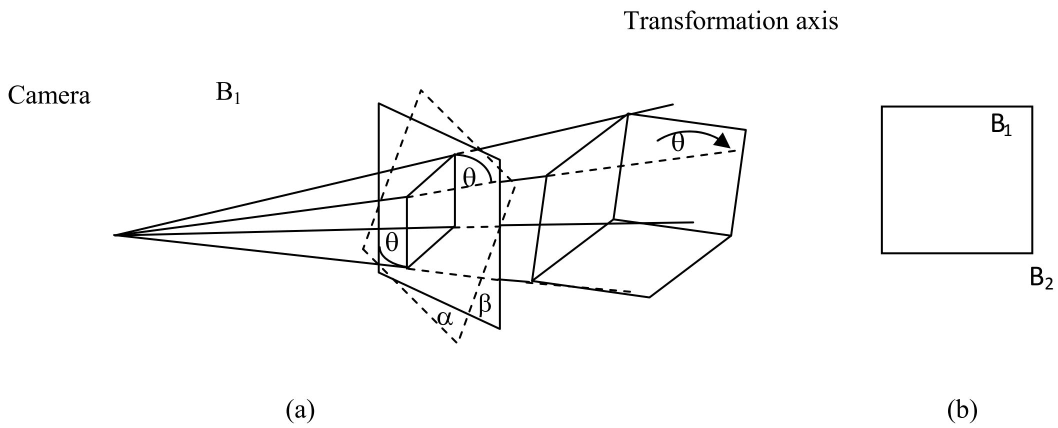

Figure 6. Scheme of occurrence of perspective distortion:
(a) Reception of the perspective image of object;
(b) Example of perspective distortion on a plane.

In the above considered case, the received images of object will be differed by parameter of perspective transformation to a certain angle of turning, and further they can pass in other quantity of images (the object is turned to the camera by other side).

In classical geometry [19, 20] such perspective transformation is named as the parabolic homology. In this case the vision of mathematical model is represented such as [18], [19], [20]:

$$B_0(x,y) = B\left(\frac{x}{\mu x + \lambda y + 1}, \frac{y}{\mu x + \lambda y + 1}\right), \tag{10}$$

where, μ, λ – Parameters of homology along the abscissa and ordinates accordingly.

Mathematical model (10) is more difficult transformation, than the models (2)–(9) because, unlike them, it is described by nonlinear group of transformations. Transformations are divided on parabolic homology along the ordinate axis P_y, where ($\mu = 0$ [19], [20]), abscisses P_x, where ($\lambda = 0$ [19], [20]) and simultaneous transformations (P_{xy}) [19], [20].

Matrixes of these transformations have the following view [18], [19], [20]:

$$P_y = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & \lambda & 1 \end{pmatrix}, \quad P_x = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \mu & 0 & 1 \end{pmatrix}, \quad P_{xy} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ \mu & \lambda & 1 \end{pmatrix},$$

Thus, the above material allows speaking about the expediency of use of the device of the group theory for the description of geometrical distortions of images. At the same time in any considered groups above as operation we understand a composition of transformation or multiplication of the matrixes corresponding to these transformations. Proceeding from properties of group there is also a return element which is capable to translate the image deformed by geometrical transformations to the reference picture. Hence, normalization process can be also described in terms of the device of the group theory.

V. NORMALIZATION OF IMAGES OF AN OBJECTIVE BY GROUP THEORY

Let some group of transformations "G" and quantity of the images $W(B_0)=\{B_1, B_2, ...B_s,\}$ received from the reference picture B_0 by transformations $g \in G$ is specified.

Since all images (B_i, $i=\overline{1,s}$) were received as a result of application of some transformation g, then according to the group theory [18, 20], there is an existence of $g \in G$ at which $gB_1 = B_2$. Then images B_1, B_2 are equivalent concerning group of transformations "G" and also concern to one class of equivalence which is presented by the reference picture B_0.

The classes of equivalence concerning the specified group correspond to different not equivalent reference pictures.

As operation that is specified in some group "G" as it has been noted above, we understand a composition of transformations or multiplication of the matrixes corresponding to these transformations.

If we will divide all quantity of input images into equivalent classes W_k concerning the chosen group of transformations, then for the recognition of the input image it is necessary to determine to what kind of equivalent class it is concerned. This is a decision of the problem of image normalization. However for this purpose it is necessary to determine a type and to find unknown parameters of transformation (1) which connects input and reference pictures.

Hence, such method of normalization consists in definition of parameters of concrete transformation $g_i \in G$ which brings the input picture to the standard:

$$B_0(x,y) = g_i[B_i(x,y)],\ i=\overline{1,s} \tag{11}$$

Normalization can be technical and algorithmic. Technical normalization is characterized by the changing of parameters of technical perception system (a focal length, position of image detector and so on) as a result, the received image of observable object will be changed too. Other way of normalization consists in transformation of the received image without changing of technical parameters of perception system. The algorithmic method of normalization allows using practically any transformations in the course of normalization, unlike a technical method which is usually limited by transformations of shifts, turnings and homogeneous scales.

Thus, the basic stage of normalization is a definition of parameters of transformation. On the basis of the found parameters $g \in G$ it is possible to construct the certain operator by means of formula:

$$F(B(x,y)) = gB(x,y) = B_0(x,y). \tag{12}$$

The operator (12) which brings the input picture to a standard view is called normalizer (concerning action of groups). The main principles of construction of such normalizer are specified below.

From definition of normalizer (12) follows, that for any images $B_1 \in W$ and $B_2 \in W$ it is fair $F(B_1)=F(B_2)$ that gives the chance of construction of normalizer without definition of some reference picture.

Then construction of normalizer defines a problem of construction of display $\Phi: W \to G$ that is for everyone $B_i \in W$, we find an element $g_i \in G$ that $F(B) = gB$. Differently, display Φ should be satisfied by a formula: $\Phi(gB) = \Phi(B)g^{-1}$.

For group "G" and a quantity of input pictures "W" the uniqueness condition is carried out: if, $gB = B$ then $g = e$ – is a unit of group. Then equality is carried out $(\Phi(B))^{-1}\Phi(gB)g = e$ it follows that group unit "G" is transformation $\Phi(B_0)$ therefore $\Phi(B) = g^{-1}$ providing definition of unknown parameters of transformation of group "G" by display construction $\Phi: W \to G$.

VI. MECHANISM AND CONSTRUCTION PRINCIPLE OF NORMALIZATION FOR IMAGE ANALYSIS

Thus, for normalizer construction for full projective group it would be necessary to find a functional which could define 9 parameters in expression (1). The solution of such problem is quite difficult, generally the insoluble. Therefore it is expedient, to divide a task into a number of more simple subtasks. For example, to carry out normalisation stage by stage, it means to influence on the input picture consistently by means of some normalizers which would bring the input picture to the standard. Such approach is concerned to a problem of construction of consecutive normalizers or normalization of consecutive type [18]. We will consider situations when such approach is comprehensible.

Let it is possible to present the group of transformations "G" in the form of composition of the subgroups ($G = G_1G_2...G_k$). Then, if "F" is a normalize "G", and "F_i" is a normalizer of groups, G_i, $i = \overline{1,k}$ then for a normalizer the synthesis conditions are carried out $F = F_1F_2...F_k$.

However at construction of consecutive normalizers, it is necessary to allocate some conditions of synthesis of normalizers, if [18]: – For any image, $B \in W$ and any transformation $g_1 \in G_1$ there is such transformation $h_1 \in G_1$, then

$$F_2(g_1B) = h_1F_2(B) \text{ or } \Phi_2(g_1B) = h_1\Phi_2(B)g_1^{-1}, \text{ at } k = 2; \tag{13}$$

The normalizers $F_1F_2...F_k$ corresponding to $G_1G_2...G_k$ are interchangeable, then $F = F_1F_2...F_k$ is normalizers of group $G = G_1G_2...G_k$;

Set of standards W_i concerning a normalizer F_i (F_i, a normalizer of group G_i) is invariant concerning group action $G = G_1G_2...G_{j-1}$ ($j = 2,...,k$), then $F = F_1F_2...F_k$ is a normalizer of group $G = G_1G_2...G_k$;

For any image $B \in W$ and any transformations $g_1 \in G_1,...,g_{k-1} \in G_{k-1}$ there will be a transformation, $h_1 \in G_1$, that, $F_2...F_k(g_{k-1}...g_1B) = h_1F_2...F_k(B)$ then $F = F_1F_2...F_k$ is a normalizer of group $G = G_1G_2...G_k$.

The resulted principles of construction of consecutive normalizers allow to synthesize them on the basis of decomposition of difficult groups of transformations, in particular, affine and projective.

However the projective group of transformations includes affine group which also has multiparametric transformations. Therefore, we will also consider a principle of construction of consecutive normalization within the limits of this group, i.e. normalization of distortions on the basis of base normalizers (as a rule, the base normalizers are one-, two-parametrical perceptions working within the limits of models (4-10) [18]). For example, one of the general approaches is transition from affine model of vision to centroaffine model. For this purpose we use invariant property of the centre of gravity concerning affine group and transform from six-parametrical affine model of vision (2) to four- parametrical models (3) by means of a partial (intermediate) normalizer of F_C type:

$$F_C(B) = B(x + \Phi_1,\ y + \Phi_2), \tag{14}$$

where Φ_1 and Φ_2 can be the centre of gravity:

$$\Phi_1 = \frac{\iint_D xB(x,\ y)dxdy}{\iint_D B(x,\ y)dxdy}, \quad \Phi_2 = \frac{\iint_D yB(x,\ y)dxdy}{\iint_D B(x,\ y)dxdy}. \tag{15}$$

After influence on the input picture with a partial normalizer F_C, we move to centroaffine model. Thus, a normalizer F_A we will present as:

$$F_A = F_{A_ц} F_C, \tag{16}$$

where $F_{A_ц}$ is a normalizer of the centroaffine group of transformations in perception models.

In turn the normalizer $F_{A_ц}$ also can be divided into more simple normalizers and the transition can be done from more difficult models of vision to less difficult.

Possibility of consecutive normalisation is based on that the full affine group of transformations is presented in the form of a composition of the elementary transformations. For example, the affine group of transformation can be distributed on the following composition of transformations [18]:

$$A = UD_{xy}U,\ A = UD_{xy}H_x,\ A = UD_{xy}H_y, \tag{17}$$

where, U –Group of turning,

D_{xy} – Group of non-uniform scale,

H_x – Group of cross shift along abscissa,

H_y – Group of cross shift along ordinate axis.

One of variants in construction of consecutive normalization of projective distortions can be the finding of the partial normalizer providing transition from projective to affine group of transformations. For this purpose it is necessary to take into account the differences of affine and projective geometry and to find the connecting link which properties would allow to construct an intermediate normalizer.

Other variant can be the finding of centering in projective geometry. By analogy to affine geometry we will assume, that there is a possibility to centre the image. In this case the model of vision becomes:

$$B_0(x, y) = B\left(\frac{b_{11}x + b_{12}y}{b_{31}x + b_{32}y + b_{33}}, \frac{b_{21}x + b_{22}y}{b_{31}x + b_{32}y + b_{33}}\right), \tag{18}$$

where transformation matrix $\Pi_ц = \begin{pmatrix} b_{11} & b_{12} & 0 \\ b_{21} & b_{22} & 0 \\ b_{31} & b_{32} & b_{33} \end{pmatrix}$ and $\det(\Pi_ц) \neq 0$.

Let's divide expression (18) into constant parameter b_{33} which is not equal to zero because $\det(\Pi) \neq 0$ according to condition of existence of considered transformation. As a result corresponding mathematical model considering analogy of transition to model (2) will become:

$$B_0(x, y) = B\left(\frac{a_{11}x + a_{12}y}{a_{31}x + a_{32}y + 1}, \frac{a_{21}x + a_{22}y}{a_{31}x + a_{32}y + 1}\right), \tag{19}$$

and accordingly the transformation matrix will look as:

$$\Pi_ц = \begin{pmatrix} a_{11} & a_{12} & 0 \\ a_{21} & a_{22} & 0 \\ a_{31} & a_{32} & 1 \end{pmatrix},$$

where $\det(\Pi_ц) \neq 0$.

By analogy to affine geometry the model of perception (19) will be called as centreaffine. In this case the normalizer of full projective group $F_{П}$ could be presented in the form of synthesis of normalizers:

$$F_{П} = F_{П_ц} F_C , \qquad (20)$$

where $F_{П_ц}$ is a normalizer of centreprojective group.

Hence it is possible to assert, that there are at least two directions for working out of a consecutive normalizer of projective group of transformations for the purpose of the image analysis:

The first consists in a finding of the partial normalizer providing transition to affine group of transformations;

The second is in working out of procedure of a centering of projective group.

VII. Conclusion

Some significant results have been obtained from the theoretical interpretation for the study of images processing. One can draw the following conclusions on the basis of present work:

(i) The generalization and construction of the theoretical models was based on geometrical transformations of images which represent projective transformation on the basis of corresponding transition to projective group of transformations.

(ii) Geometrical interpretation of the analyzed image of object is considered in detail on the basis of use of the device of the group theory. And also various subgroups of projective group in the description of possible geometrical distortions of the analyzed image are considered for this purpose.

(iii) The generalization of normalization process of image in terms of the group theory have been demonstrated in a new valuable fashion.

(iv) Features of the separate groups of transformations are shown on the basis of consideration of corresponding matrixes of transformations. At the same time such consideration has allowed to generalize different methods and principles of construction of normalizers for the image analysis.

(v) Two directions for working out of a consecutive normalizer of projective group of transformations are offered for the purpose of reduction of span time for carrying out of normalization process of images and possibility of realization of such process as a whole.

(vi) There is shown the possibility of realization of the offered transition in the form of synthesis of difficult normalizers by means of more simple normalizers on the formalized level. It allows speaking about the possibility of the algorithmic solution of the problem of normalization for various groups of geometrical transformations of investigated images as a whole.

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