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Mathematical Modeling of Mutual Relationship and Countable Extension of Connected Nodes in Social Networking

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Abstract: ***Srivastav et al already published a paper where they have established a graph theoretical model to describe social networks. They have shown that it is possible to describe any social networks using simple nodes and edges. In the present work the authors have made further study on mathematical modeling of Social area networks in which it is shown how nodes are connected with other nodes and also explain the role of mutual relationship. The authors have correlated the 'AND' GATE LOGIC to explain the connectivity of two nodes. In this paper authors have tried to explore the method of connectivity between the nodes in different situation.***

Keywords: mutual; social; gate; connectivity; nodes; edges.

I. INTRODUCTION

The beauty of a social networking system is to connect the nodes and enlarge (extend) the relationship between two nodes. The relationships between two nodes are either public or private as the case may be. The existence of connection between two nodes can be extended to more nodes with the members of a social network. The relationship of a single node can be spanned with another node through mutual relationship process. One of the definitions of Mutual is defined as: "of or pertaining to each of two or more; held in common; shared:" So a mutual relationship would be a relationship in which the people in it have common interests/goals of the relationship. Mutual means that both people feel the same way. For example if suppose "node X wants to friend of node Y" and node Y also feels similar and says "The feeling is mutual", that means Y is friend of X. The aim of this type of connection is to prepare the relationship between maximum nodes in the optimal time, optimal cost and optimal number of edges that is required to connect.

II. MATHEMATICAL DESCRIPTION OF CONNECTIVITY OF NODES IN SOCIAL NETWORK

1. *Social network* [1,2]

A social network is a social structure made up of individuals (or organizations) called "nodes", which are tied (connected) by one or more specific types of interdependency, such as friendship, kinship, common interest, financial exchange, likings or disliking, or relationships of beliefs, knowledge or prestige. Social network analysis views social relationships in terms of network theory consisting of nodes and ties (also called edges, links, or connections). Nodes are the individual actors within the networks, and ties are the relationships between the actors.

Hence, mathematically, social networking can be defined as the collection of socially connected elements/objects. i.e, set

$$S = \{ \text{social elements} : \text{social elements are connected} \}$$

In a social network S, consider two nodes (i)node1 represented by N_1 and(ii)node2 represented by N_2 and consider a function f defined between node N_1 and node N_2

$$\text{i.e., } f : N_1 \rightarrow N_2$$

such that message send by node N_1 is received by node N_2

i.e, if $m_1,m_2,m_3……..m_n$ be the message send by the node N_1 to N_2,then $f(m_1)$, $f(m_2)$,$f(m_3)$,……..$f(m_n)$ will be message received by N2.

$$\text{i.e., } \{m_1,m_2,m_3……..m_n\} \rightarrow \{ f(m_1), f(m_2) ,f(m_3),……..f(m_n)\}$$

$$\text{i.e, set of send message by node } N_1 \rightarrow \text{ set of received message by node } N_2.$$

Definition1 [3]

A social network is modeled as a graph G(V, E), where V represents a set of users embedded in a social context, and the edge set E={(x, y)| x, y ∈V} represents friendship among users . An edge e = (x, y) is added to E when a friend request from x to y or from y to x is accepted. In SNSs(social networking sites), such as Facebook and LinkedIn, edges are usually undirected. For each user u (u ∈V), the set F(u)={ x | x ∈V, (x,u)∈E}represents the friend list of u. Note, for each edge e = (x, y), x ∈ F(y) and y ∈F(x).

The mutual friends between two users can be defined as follows:

Definition 2 [3]

Given two users x and y (x, y ∈V), we define the set of the mutual friends, MF(x, y),between them as MF(x, y) = F(x) ∩ F(y).Here, MF(x, y) stands for mutual friendship between x and y,

Intuitively, the definition of the mutual friend has two properties:

» Given x, y, z ∈ V, y ∈ MF(x, z) ⇔ y ∈ F(x) or y ∈F(z).

» Given x, z, MF(x, z) = MF(z, x); i.e., mutual friendship is irreflexive and symmetric.

2.2 To understand the basic method of connectivity between two nodes of social network

Let us consider the two set of nodes S_1 and S_2

$$S_1= \{n_1,n_2,n_3,n_4,n_5\} \text{ and } \quad S_2= \{m_1,m_2,m_3,m_4,m_5, m_6,m_7,m_8\} \text{ of a social network.}$$

Suppose, nodes n_1 wants to connect with maximum number of members of $\{m_1,m_2,m_3,m_4,m_5, m_6,m_7,m_8\}$.So, node n_1 can send friend request to any member of set S_2.

Step1 : Start the connectivity between nodes.

Step2 : Let, node n_1 send first friend request to node m_1. Node m_1 has option to accept the friend request or can reject the friend request.

(i) If m_1 reject the connection of n_1 then n_1 has to option to make connection with $\{m_2,m_3,m_4,m_5, m_6,m_7,m_8\}$ as stated in step 1.

(ii) If m_1 accept friends request then there exist a connectivity(connected edge) between n_1 and m_1

$$n_1 \text{ ——— } m_1$$

Figure: 1

Step3: When a node n_1 of Social network is being connected to another node m_1 of social network then m_1 will acts as a parent node(root node) and all other related node will acts as branch node .i.e., there is some member from this set $\{m_2,m_3,m_4,m_5, m_6,m_7,m_8\}$ will acts branch member. A parent node have either no branch node or greater than or equal to one branch node. Suppose m_2, m_3 are branch nodes of nodes m1.

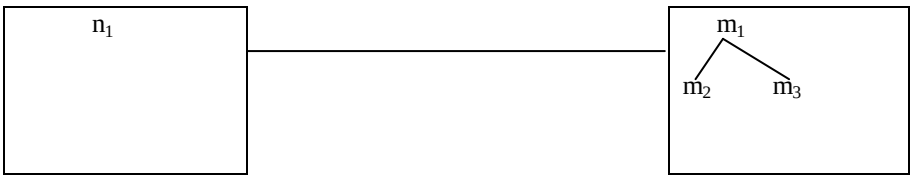

Figure: 2

Step4: node m_2 or m_3 may be publically or privately connected with nodes with nodes m_1.The only public nodes are seen by the nodes n_1 and so n_1 has option to make connection with the public nodes m_1.

Step 4.1 Let, nodes m_2 is public with nodes m_1 :

In this case all public related nodes of m_2 will be seen by the nodes n_1 and m_2 will acts as root nodes. Hence n_1 has an option to make connection between the publically related nodes of m_2. Here, a mutual relationship may be established between the nodes n_1 and m_2 if both of them like to establish the relation to each other.

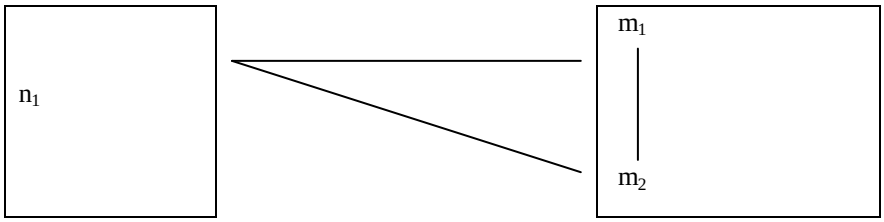

Figure : 3

Hence, $\{n_1,m_1,m_2\}$ are mutually connected nodes in social network.

Step 4.2 Let, nodes m_3 is private with the nodes m_1:

In this case all m_3 and its related nodes of m_3 will not be seen by the nodes n_1.Hence n_1 can not make the relationship with m_3 and its branch nodes m3.

Hence, overall situation look like this after this step as:

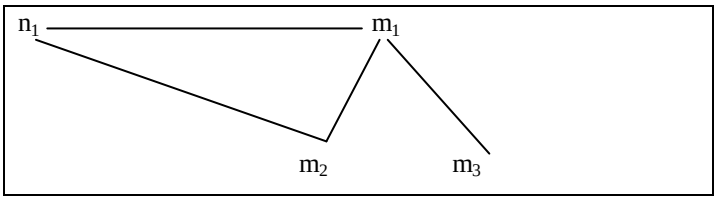

Figure:4

Step5 : Repeat the process of step1 to step4 as per requirement.

(i)suppose,$\{m_4,m_5,m_6\}$ are branch nodes of m_2.i.e,

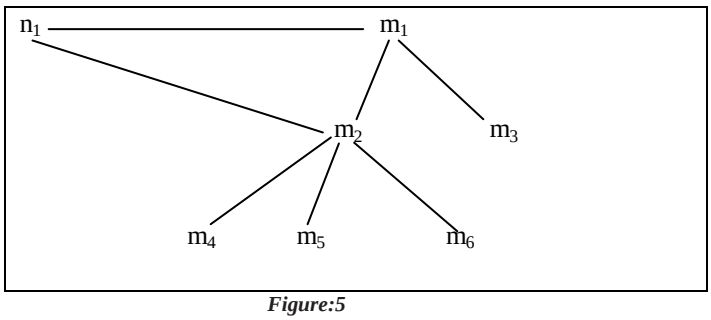

Figure:5

then it is required to check the step 4, step 4.1 and step 4.2.

(ii) suppose $\{m_7,m_8\}$ are branch nodes of m_3 then there is not an option for n_1 to make connection with nodes $\{m_6,m_7,m_8\}$at a time being since m_3 is privately related with m_1

Step 6: Stop the process of connection after all the nodes are checked and make connection with likely (possible) nodes.

Redundancy of relationship

Redundancy means the state of being not or no longer needed or useful. In redundancy there is a provision or existence of more than one means or resources to perform an activity or function. Redundancy of relationship may consist of repeated number of nodes, edges etc. When one set of nodes want to make connection or relation with another set of node in the social networking there may be chances of repeated nodes and edges because social networking follows the rules of mutual relationship.

III. AND GATE AND ITS CO-RELATION WITH 'RELATIONSHIP OF NODES' IN SOCIAL NETWORK

The relationship for the connectivity between the two follows the same theory of AND gate of Logic gate

(i) AND gate

Figure:6

2 Input AND gate		
A	B	A.B
0	0	0
0	1	0
1	0	0
1	1	1

Figure:7

The AND gate is an electronic circuit that gives a **high** output (1) only if **all** its inputs are high. A dot (.) is used to show the AND operation i.e. A.B. Bear in mind that this dot is sometimes omitted i.e. AB

(ii) Social network relationship with respect to nodes in sense of logic AND gate

Suppose two nodes want to make connection between two nodes then the following cases may be occur:

Connectivity between two nodes		
Node N_1	Node N_2	Result/Remark
Node N_1 do not like to make connection with N_2	Node N_2 do not like to make connection with N_1	There does not exist connection between the two nodesN_1 and N_2
Node N_1 do not like to make connection with node N_2	Node N_2 like to make connection with node N_1	There does not exist connection between the two nodes N_1 and N_2
Node N_1 like to make connection with node N_2	Node N_2 do not like to make connection with node N_1	There does not exist connection between the two nodes N_1 and N_2
Node N_1 like to make connection with node N_2	Node N_2 like to make connection with node N_1	Exist connection between the two nodes N_1 and N_2

Table:1

(iii)To understand connection between node N_1, node N_2, branch node N_3 of N_2

When a node N_1 of Social network is being connected to another node N_2 of social network then N_2 will acts as a parent node(root node) and all other related node will acts as branch node. Here N_3 is working as branch node.

<table>
<tr><th>Node N_1</th><th>Node N_2
(Note: Node N_2 is taken as root node of social network S_2)</th><th>Relationship between node N_1 and N_2</th><th colspan="2">Node N_3
(Note: Node N_3 is taken as branch node of node N_2)</th><th>Relationship between node $N_1$and N_3</th></tr>
<tr><td>Node N_1 do not like to make connection with node N_2</td><td>Node N_2 do not like to make connection with node N_1</td><td>There does not exist connection between the two nodes N_1 and N_2</td><td colspan="2">Since root node N_2 do not like to make connection with node N_1.So, at the time being node N_3 can not make the connection with N_1 via. the path of node N_2.</td><td>There does not exist connection between the two nodes N_1 and N_3.</td></tr>
<tr><td>Node N_1do not like to make connection with node N_2</td><td>Node N_2 like to make connection with node N_1</td><td>There does not exist connection between the two nodesN_1 and N_2</td><td colspan="2">Since root node N_1 do not like to make connection with node N_2.So, at the time being node N_3 cannot make the connection with N_1 via. the path of N_2</td><td>There does not exist connection between the two nodes N_1 and N_3</td></tr>
<tr><td>Node N_1 like to make connection with node N_2</td><td>Node N_1 do not like to make connection with N_1</td><td>There does not exist connection between the two nodes</td><td colspan="2">Since root node N_2 do not like to make connection with node N_1.So, at the time being N_3 cannot make the connection with N_1 via. The path of node N_2.</td><td>There does not exist connection between the two nodes N_1 and N_3</td></tr>
<tr><td rowspan="6">Node N_1 like to make connection with node N_2</td><td rowspan="6">Node N_2 like to make connection with N_1</td><td rowspan="6">Exist connection between the two nodes</td><td colspan="2">NodeN_3 can be either public or private with Node N_1.
CASE1: WHEN NODE N_3 IS PUBLIC WITH NODE N_2</td><td rowspan="2">There does not exist connection between the two nodes N_1 and N_3</td></tr>
<tr><td>Node N_1</td><td>Node N_3</td></tr>
<tr><td>Node N_1 do not like to make connection with N_3</td><td>Node N_3 do not like to make connection with N_1</td><td></td></tr>
<tr><td>Node N_1 do not like to make connection with N_3</td><td>Node N_3 like to make connection with N_1</td><td>There does not exist connection between the two nodes N_1 and N_3</td></tr>
<tr><td>Node N_1like to make connection with node N_3</td><td>Node N_3 do not like to make connection with N_1</td><td>There does not exist connection between the two nodes N_1 and N_3</td></tr>
<tr><td>Node N_1like to make connection with N_3</td><td>Node N_3 like to make connection with N_1</td><td>There does not exist connection between the two nodes N_1 and N_3</td></tr>
<tr><td colspan="2">CASE2: WHEN NODE N_3 IS PRIVATE WITH NODE N_2.</td><td>There does not exist connection between the two nodes N_1 and N_3 at the time being.</td></tr>
</table>

Table:2

IV. Countable Nodes In Social Networking

Definition:(Countable Set)

A set A is said to be finite, if A is empty or there is n ∈ N ,set of Natural Number, and there is a bijection f : {1, . . . , n} → A. Otherwise the set A is called infinite. Two sets A and B are called equinumerous, written A ∼ B, if there is a bijection f : X → Y . A set A is called countably infinite if A ∼ N. We say that A is countable if A ∼ N or A is finite.

In general, the unknown nodes in the social network are connected by searching the name, place or using different characteristics of nodes . In this method it is helpful for a node to extends its connectivity with more nodes by using mutual friendship relation .The number of connected nodes and number of connected edge are making one to one correspondence with set of Natural number. Hence, number of connected nodes and number of connected edge are either finite or countable infinite.

V. Conclusion And Future Scope

This method is also helpful for the mathematical problem to prepare optimal path to make the relation/connection between two or more nodes in optimal time and optimal cost. There is an scope to reduce the redundancy of relationship in social networking.

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Mr. Manoj Kumar Srivastav has been Post graduate in Pure Mathematics from University of Calcutta in year 2004 with Special paper in advanced functional analysis and category theory, universal algebra and lattice theory. At present he is doing MCA from IGNOU at St.Xavier's college as study center and he is working as a postgraduate teacher in Mathematics in an esteemed institution. He has more than 8 published research papers in National and International Journals.

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