ISSN: 2321-7782 (Online)
Volume 3, Issue 11, November 2015
International Journal of Advance Research in Computer Science and Management Studies
Research Article / Survey Paper / Case Study
Available online at: www.ijarcsms.com

Fourier-Finite Mellin Transforms of Some Special Functions

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Abstract: Linear transforms, especially those named for Fourier and Mellin are well known as providing techniques for solving problems in linear systems. Characteristically, one uses the transformation as a mathematical or physical tool to alter the problem into one that can be solved. The main aim of this paper is to find the Fourier-Finite Mellin transforms of some special functions and this will be used for solving various differential and integral equations.

Keywords: Fourier transform; Finite Mellin transform; Fourier- Finite Mellin transform; Integral Transform; Linear Transform.

I. INTRODUCTION

Transform methods provide a unifying mathematical approach to the study of electrical networks, devices for energy-conversion and control, antennas, and other components of electrical systems, as well as to complete linear systems and to many other physical systems and devices, whether electrical or not. These same methods apply equally to the subjects of Electrical Communication by wire or optical fiber, to wireless radio propagation, and to ionized media-which are all concerned with the interconnection of electrical systems-and to information theory which, among other things, relates to the acquisition, processing, and presentation of data. Despite the strong bonds with electrical engineering, Fourier analysis nevertheless has become indispensable in bio-medicine and remote sensing (geophysics, oceanography, planetary surfaces, civil engineering), where practitioners now outnumber those electrical engineering who regularly use Fourier analysis [1].

The Mellin Transform is widely used in Computer Science because of its scale invariance property. The magnitude of the Mellin Transform of a scaled function is identical to the magnitude of the original function. This scale invariance property is analogous to the Fourier Transform's shift invariance property. The magnitude of a Fourier transform of a time-shifted function is identical to the original function [2]. The Mellin transform is used in analysis of the prime-counting function and occurs in discussions of the Riemann zeta function. Inverse Mellin transforms commonly occur in Riesz means. The Mellin transform can be used in Audio timescale-pitch modification [3].

These Fourier and Mellin transforms have various uses in many fields separately. On combining these two transforms i.e. Fourier-Finite Mellin transforms also used for solving differential and integral equations. In this paper we find the Fourier-Finite Mellin transforms of some special functions which is help for solving differential equations.

This paper is planned as follows:

Preliminary results are given in section 2. In section 3, we have find the Fourier-Finite Mellin transforms of some special functions. Lastly conclusions are given in section 4.

Notations and terminology as per Zemanian. [4], [5].

II. PRELIMINARY RESULTS

The Fourier transform with parameter s of $f(t)$ denoted by $F[f(t)]=F(s)$ and is given by

$$F[f(t)]=F(s)=\int_{-\infty}^{\infty}e^{-ist}f(t)dt$$, for parameter $s>0$. (2.1)

The Finite Mellin Transform with parameter p of $f(x)$ denoted by $M_f[f(x)]=F(p)$ and is given by

$$M_f[f(x)]=F(p)=\int_0^a\left(\frac{a^{2p}}{x^{p+1}}-x^{p-1}\right)f(x)dx$$, for parameter $p>0$. (2.2)

The Conventional Fourier-Finite Mellin transform is defined as

$$FM_f\{f(t,x)\}=F(s,p)=\int_{-\infty}^{\infty}\int_0^a e^{-ist}\left(\frac{a^{2p}}{x^{p+1}}-x^{p-1}\right)f(t,x)dtdx \tag{2.3}$$

where, $K(t,x,s,p)=e^{-ist}\left(\frac{a^{2p}}{x^{p+1}}-x^{p-1}\right)$.

III. FOURIER-FINITE MELLIN TRANSFORMS OF SOME SPECIAL FUNCTIONS

3.1. If $FM_f\{f(t,x)\}(s,p)$ denotes generalized Fourier-Finite Mellin transform of $f(t,x)$ then

$$FM_f\{1\}=\frac{2ia^p}{sp}$$

Proof:- We have

$$FM_f\{f(t,x)\}=\int_0^{\infty}\int_0^{a}e^{-ist}\left(\frac{a^{2p}}{x^{p+1}}-x^{p-1}\right)f(t,x)dtdx$$

$$FM_f\{1\}=\int_0^{\infty}\int_0^{a}e^{-ist}\left(\frac{a^{2p}}{x^{p+1}}-x^{p-1}\right)(1)dtdx$$

$$=\int_0^{\infty}e^{-ist}dt.\int_0^{a}\left(\frac{a^{2p}}{x^{p+1}}-x^{p-1}\right)dx$$

$$=\left[\frac{e^{-ist}}{-is}\right]_0^{\infty}.\left\{a^{2p}\int_0^a x^{-p-1}dx-\int_0^a x^{p-1}dx\right\}$$

$$=\frac{1}{is}\left\{a^{2p}\left(\frac{x^{-p}}{-p}\right)_0^a-\left(\frac{x^p}{p}\right)_0^a\right\}$$

$$=\frac{1}{is}\left\{-\frac{a^{2p}}{p}\left(a^{-p}\right)-\frac{1}{p}\left(a^{p}\right)\right\}$$

$$=\frac{1}{is}\left\{-\frac{a^{p}}{p}-\frac{a^{p}}{p}\right\}=\frac{1}{is}\left\{\frac{-2a^{p}}{p}\right\}=\frac{2ia^{p}}{sp}$$

$$\therefore FM_{f}\{1\}=\frac{2ia^{p}}{sp}$$

3.2. If $FM_{f}\{f(t,x)\}(s,p)$ denotes generalized Fourier-Finite Mellin transform of $f(t,x)$ then $FM_{f}\{(\delta(t-a)\delta(x-b))\}=K(a,b,s,p)$

Proof:

$$FM_{f}\{(\delta(t-a)\delta(x-b))\}=\int_{0}^{\infty}\int_{0}^{a}e^{-ist}\left(\frac{a^{2p}}{x^{p+1}}-x^{p-1}\right)\{(\delta(t-a)\delta(x-b))\}\,dtdx$$

$$=\int_{0}^{\infty}\delta(t-a)e^{-ist}dt\int_{0}^{a}\delta(x-b)\left(\frac{a^{2p}}{x^{p+1}}-x^{p-1}\right)dx$$

$$=e^{-isa}\left(\frac{a^{2p}}{b^{p+1}}-b^{p-1}\right)=K(a,b,s,p)$$

We know that $\int_{0}^{\infty}\delta(t-a)\phi(t)\,dt=\phi(a)$ Also $\int_{0}^{a}\delta(t-a)\phi(t)\,dt=\phi(a)$

$$\therefore FM_{f}\{(\delta(t-a)\delta(x-b))\}=K(a,b,s,p)$$

3.3. If $FM_{f}\{f(t,x)\}(s,p)$ denotes generalized Fourier-Finite Mellin transform of $f(t,x)$ then

$$FM_{f}\{tx\}=\frac{2pa^{p+1}}{s^{2}\left(p^{2}-1\right)}$$

Proof:

$$FM_{f}\{tx\}=\int_{0}^{\infty}\int_{0}^{a}e^{-ist}\left(\frac{a^{2p}}{x^{p+1}}-x^{p-1}\right)(tx)\,dtdx$$

$$=\int_{0}^{\infty}e^{-ist}t\,dt.\int_{0}^{a}x\left(\frac{a^{2p}}{x^{p+1}}-x^{p-1}\right)dx$$

$$=\left[\left(t\frac{e^{-ist}}{-is}\right)_{0}^{\infty}-\int_{0}^{\infty}1\frac{e^{-ist}}{-is}dt\right].\left[\int_{0}^{a}\left(\frac{a^{2p}}{x^{p}}-x^{p}\right)dx\right]$$

 ISSN: 2321-7782 (Online)

$$=\left[\frac{1}{is}\int_0^\infty e^{-ist}dt\right].\left[a^{2p}\int_0^a x^{-p}dx-\int_0^a x^p dx\right]$$

$$=\frac{1}{is}\left[\frac{e^{-ist}}{-is}\right]_0^\infty\left[a^{2p}\left(\frac{x^{-p+1}}{-p+1}\right)_0^a-\left(\frac{x^{p+1}}{p+1}\right)_0^a\right]$$

$$=\frac{1}{i^2s^2}\left[\frac{a^{2p}}{-p+1}\left(a^{-p+1}\right)-\frac{1}{p+1}\left(a^{p+1}\right)\right]$$

$$=-\frac{1}{s^2}\left[\frac{a^{p+1}}{-p+1}-\frac{a^{p+1}}{p+1}\right]$$

$$=-\frac{1}{s^2}\left[\frac{pa^{p+1}+a^{p+1}+pa^{p+1}-a^{p+1}}{(-p+1)(p+1)}\right]=\frac{-2pa^{p+1}}{s^2\left(1-p^2\right)}=\frac{2pa^{p+1}}{s^2\left(p^2-1\right)}$$

$$\therefore FM_f\{tx\}=\frac{2pa^{p+1}}{s^2\left(p^2-1\right)}$$

3.4. If $FM_f\{f(t,x)\}(s,p)$ denotes generalized Fourier-Finite Mellin transform of $f(t,x)$ then

$$FM_f\{t^n x^n\}=\frac{(-1)^{-(n+1)}(i)^{-(n+1)}2n!\,pa^{n+1}}{s^{n+1}\left(n^2-p^2\right)}$$

Proof:

$$FM_f\{t^n x^n\}=\int_0^\infty\int_0^a e^{-ist}\left(\frac{a^{2p}}{x^{p+1}}-x^{p-1}\right)\left(t^n x^n\right)dtdx$$

$$=\int_0^\infty e^{-ist}t^n dt.\int_0^a x^n\left(\frac{a^{2p}}{x^{p+1}}-x^{p-1}\right)dx$$

$$=\left[\left(t^n\frac{e^{-ist}}{-is}\right)_0^\infty-\int_0^\infty nt^{n-1}\frac{e^{-ist}}{-is}dt\right].\left[a^{2p}\int_0^a x^{-p+n-1}dx-\int_0^a x^{p+n-1}dx\right]$$

$$=\left[\frac{n}{is}\int_0^\infty t^{n-1}e^{-ist}dt\right].\left[a^{2p}\left(\frac{x^{-p+n}}{-p+n}\right)_0^a-\left(\frac{x^{p+n}}{p+n}\right)_0^a\right]$$

$$=\frac{n}{is}\left[\left(t^{n-1}\frac{e^{-ist}}{-is}\right)_0^\infty-\int_0^\infty(n-1)t^{n-2}\frac{e^{-ist}}{-is}dt\right].\left[\frac{a^{2p}}{-p+n}a^{-p+n}-\frac{a^{p+n}}{p+n}\right]$$

 ISSN: 2321-7782 (Online)

$$=\left[\frac{n}{is}\frac{(n-1)}{is}\int_0^\infty t^{n-2}e^{-ist}dt\right].\left[\frac{a^{p+n}}{-p+n}-\frac{a^{p+n}}{p+n}\right]$$

$$\vdots$$

$$\vdots$$

$$=\left[\frac{n(n-1)(n-2)(n-3)\cdots\cdots 2.1}{(is)^n}\int_0^\infty e^{-ist}dt\right]\left[\frac{pa^{p+n}+na^{p+n}+pa^{p+n}-na^{p+n}}{n^2-p^2}\right]$$

$$=\frac{2n!\,pa^{p+n}}{(is)^{n+1}(n^2-p^2)}=\frac{(-1)^{-(n+1)}(i)^{-(n+1)}2n!\,pa^{p+n}}{(s)^{n+1}(n^2-p^2)}$$

$$\therefore FM_f\{t^n x^n\}=\frac{(-1)^{-(n+1)}(i)^{-(n+1)}2n!\,pa^{p+n}}{s^{n+1}(n^2-p^2)}$$

Fourier-Finite Mellin Transforms of Some Elementary Functions

Sr. No	$f(t,x)$	$FM_f\{f(t,x)\}=F(s,p)$
1	1	$\frac{2ia^p}{sp}$
2	$\delta(t-a)\delta(x-b)$	$e^{-isa}\left(\frac{a^{2p}}{b^{p+1}}-b^{p-1}\right)=K(a,b,s,p)$
3	tx	$\frac{2pa^{p+1}}{s^2(p^2-1)}$
4	$t^n x^n$	$\frac{(-1)^{-(n+1)}(i)^{-(n+1)}2n!\,pa^{n+1}}{s^{n+1}(n^2-p^2)}$
5	e^{at+bx}	$\frac{(-1)^{-p+1}b^{-p}\left[(-ab)^{2p}(\Gamma(-p,-ab)-\Gamma(-p))-(\Gamma(p,-ab)-\Gamma(p))\right]}{(is-a)}$
6	$\sin at\sin bx$	$\frac{ia(-ib)^{-p}}{2(a^2-s^2)}\{(iab)^{2p}\left[(\Gamma(-p,-iab)-\Gamma(-p))-(-1)^p(\Gamma(-p,iab)-\Gamma(-p))\right]$ $+\left[(\Gamma(p,-iab)-\Gamma(p))+(-1)^p(\Gamma(p,iab)-\Gamma(p))\right]\}$
7	$\cos at\cos bx$	$\frac{sb^{-p}(-i)^{-p+1}}{2(a^2-s^2)}\{(iab)^{2p}\left[(\Gamma(-p,-iab)+\Gamma(-p))-(-1)^p(\Gamma(-p,iab)-\Gamma(-p))\right]$ $-\left[(\Gamma(p,-iab)-\Gamma(p))+(-1)^p(\Gamma(p,iab)-\Gamma(p))\right]\}$

8	$\sinh(at)\sinh(bx)$	$\frac{ab^{-p}(-1)^{-p+1}}{2(s^2+a^2)}\Big\{(ab)^{2p}\Big[(-1)^p\big(\Gamma(-p,ab)-\Gamma(-p)\big)-\big(\Gamma(-p,-ab)-\Gamma(-p)\big)\Big]$ $-\Big[(-1)^p\big(\Gamma(p,ab)-\Gamma(p)\big)-\big(\Gamma(p,-ab)-\Gamma(p)\big)\Big]\Big\}$
9	$\cosh(at)\cosh(bx)$	$\frac{is(-b)^{-p}}{2(s^2+a^2)}\Big\{(ab)^{2p}\Big[\big(\Gamma(-p,-ab)-\Gamma(-p)\big)+(-1)^p\big(\Gamma(-p,ab)-\Gamma(-p)\big)\Big]$ $-\Big[\big(\Gamma(p,-ab)+\Gamma(p)\big)+(-1)^p\big(\Gamma(p,ab)-\Gamma(p)\big)\Big]\Big\}$

IV. Conclusion

In the present paper the Fourier-Finite Mellin transforms of some special functions are obtained and this will be used for solving various differential and integral equations.

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ISSN: 2321-7782 (Online)