ISSN: 2321-7782 (Online)
Volume 2, Issue 9, September 2014
International Journal of Advance Research in Computer Science and Management Studies
Research Article / Survey Paper / Case Study
Available online at: www.ijarcsms.com

To Find a 2 – Tuple Dominating Set of an Induced Subgraph of a Split Restrained Dominating Set of an Interval Graph Using an Algorithm

Dr. A.Sudhakaraiah[1]
Department of Mathematics
S.V.University, Tirupati
Andhra Pradesh – India

K.Ramakrishna[2]
Department of Mathematics
S.V.University, Tirupati
Andhra Pradesh – India

Dr. E.Gnana Deepika[3]
Department of Mathematics
S.V.University, Tirupati,
Andhra Pradesh – India

Abstract: In Graph Theory, a connected component of an undirected graph is a sub graph in which any two vertices are connected to each other by paths. For a graph G, if the sub graph G itself is a connected component, then the graph G is called connected, else the graph G is called disconnected and each connected component sub graph is called its components. A dominating set D of graph G (V, E) is a split dominating set, if the induced sub graph < V – D> is disconnected. The split dominating number $\gamma_s(G)$ of G is the minimum cardinality of a split dominating set. The 2 – tuple domination problem is to find a minimum size vertex subset such that every vertex in the graph is dominated by at least 2 vertices in the set. A set D is called a restrained dominating set, if every vertex in V(G) – D is adjacent to a vertex in D and another vertex in V(G) – D. In this paper we discussed an algorithm to find a 2 – tuple dominating set of an induced sub graph of a split restrained dominating set of an interval graph.

Key Words: Interval family, Interval graph, connected graph, Dominating Set, Split dominating set, 2 – tuple domination, restrained dominating set and design of an algorithm.

I. INTRODUCTION

An undirected graph G = (V, E) is an interval graph (IG), if the vertex set V can be put into one-to-one correspondence with a set of intervals I on the real line R, such that two vertices are adjacent in G, if and only if their corresponding intervals have non-empty intersection. The set I is called an interval representation of G and G is referred to as the intersection graph I. Let $I = \{I_1, I_2, I_3, \ldots\ldots, I_n\}$ be any interval family, where each I_i is an interval on the real line and $I_i = [a_i, b_i]$, for i = 1, 2, 3,......,n. Here a_i is called the left end point labelling and b_i is the right end point labelling of I_i. Without loss of generality we assume that all end points of the intervals of the intervals in I are distinct numbers between 1 and 2n. Two intervals i and j are said to be intersect each other if they have non-empty intersection. Also we say that the intervals contain both its end points and that no two intervals share a common end point. The intervals and vertices of an interval graph are one and the same thing. The graph G is connected and the list of sorted end point is given and the intervals in I are indexed by increasing right end point, that is $b_1 < b_2 < b_3 < \ldots\ldots < b_n$.

Let G(V, E) be a graph, a set S is a dominating set of, if every vertex in < V – D > is adjacent to some vertex in D. The domination number $\gamma_c(G)$ of G is the minimum cardinality of a connected dominating set. A dominating set D of a graph G(V,

E) is a split dominating set if the induced sub graph <V – D > is disconnected. The split domination number $\gamma_s(G)$ of G is the minimum cardinality of a split dominating set.

In a graph G, a vertex is said to dominate itself and its entire neighbour [5]. A dominating set of G (V, E) is a subset D of V such that every vertex in V is dominated by at least one vertex in D. The domination number $\gamma(G)$ is the minimum size of a dominating set of G. For a fixed positive integer m an m-tuple dominating set of G(V, E) is a subset D of V such that every vertex in V is dominated by at least m vertices of D. As introduced by Harary and Haynes [2, 4] a m-tuple dominating set D is a set $D \subseteq V$ for which $|N[v] \cap D| \geq m,$ for every $v \in V,$ Where $N[v] = \{v\} \cup \{u \in V : (u,v) \in E\}$ is the closed neighbourhood of the vertex v. A double dominating set D is said to be minimal if there does not exist any $D'' \subset D$ such that D'' is a double dominating set of G. A double dominating set D, denoted by $\gamma_{x2}(G)$ is said to be minimum, if it is minimal as well as it gives double domination number [3]. A set D is called a restrained dominating set, if every vertex in V(G) – D is adjacent to a vertex in D and another vertex in V(G) – D [2]. The purpose of this paper is to find the 2 – tuple dominating set of an induced sub graph a split dominating set by introducing the Algorithm.

Main Theorems:

Theorem 1: Let D be a dominating set of given interval graph G. If I_i and I_j are any two intervals in I such that $I_i \in RDS$, $I_j \neq 1$ and I_j is contained in I_i and if there is atleast one interval to the left of I_j that intersects I_j and there is no interval $I_k \neq I_i$ to the right of I_j that intersects I_j. Then Split restrained domination occurs in G and also the cardinality of the double dominating set of the disconnected induced sub graph G_1=<V-RDS> is $|D_1| = n_1 - 1,$ Where n_1 is the number of vertices of G_1.

Proof: Let $I = \{I_1, I_2, I_3, ..., I_m\}$ be the given interval family and G is an Interval graph corresponding to I. Let I_i and $I_j \neq 1$ be any two intervals in I such that I_j is contained in I_i. Suppose that RDS be a restrained dominating set of G and suppose $I_i \in RDS$.

Let I_l be an interval in I, which is to the left of I_j and intersects I_j. Further by our assumption there is no interval $I_k \neq I_i$ to the right of I_j that intersects I_j. Let us assume that there is at least one interval in the set $I = \{I_{i+1}, I_{i+2}, I_{i+3}, ..., I_m\}$ which intersects I_j, then we let cannot get the split domination as I_l is an interval in I, which is to the left of I_j and intersects I_j. Hence there is no interval to the right of I_j that intersect I_j. If such an interval exists then there must be any interval to the left of I_j that intersects I_j. Again we have to compute the double dominating set of an induced sub graph of split restrained dominating set and then show that $|D_1| = n_1 - 1,$ where n_1=7 the number of vertices in the split restrained dominating set and D_1 is the double dominating set of induced sub graph of split restrained dominating set of an interval graph. Next we will find the 2-tuple dominating set of an induced sub graph <V-RDS> by using an algorithm as follows.

An algorithm for finding 2-tuple domination:

Input: An interval graph $G = (V, E)$ with IG ordering vertex set $V = \{1, 2, 3, ..., n\}$.

Output: 2-tuple dominating set D_1.

Step 1: Set $f(j) = 0, \forall j = 1, 2, ..., n;$ // Assume that no vertices are the members of D_1.//

Step 2: Set $i = 1, D_1 = \phi$.

Step 2.1: Compute $W_i(f) = \sum_{v \in N[i]} f(v)$; $(v \in V(G))$;

Step 2.1.1: If $W_i(f) = 0$ then // At least the vertex i is not adjacent to any of the vertices of D_1. //

Step 2.1.1.1: $f(M_m(i)) = 1$ and $f(M_{m+1}(i)) = 1$;

Where m is the index of the starting neighbourhood of i in the table of p^{th} numbered adjacent vertices.

$$D_1 = D_1 \cup \{M_m(i)\} \cup \{M_{m+1}(i)\}$$

Step 2.1.1.2: Else set $f(M_m(i)) = 1$, if m is the only neighbourhood of i in the table of p^{th} numbered adjacent vertices.

Then $D_1 = D_1 \cup \{M_m(i)\}$

Step 2.1.2: Else if W^i (f)=1 then // at least the vertex i is connected to one of the vertex of D_1//

Step 2.1.2.1: If $f(M_{m_1}(i)) = 1$, then set $f(M_{m-1}(i)) = 1$,

Where m_1 is the index of the ending neighbourhood of i

$$D_1 = D_1 \cup \{M_{m-1}(i)\}$$

Step 2.1.2.2: Else set $f(M_{m_1}(i)) = 1$

$$D_1 = D_1 \cup \{M_{m_1}(i)\}$$

End if

Else

Go to Step 2.2;

End if;

Step 2.2: Calculate $i = i+1$ and go to Step 2.1 and continue until $i>n$;

End algorithm.

Illustration: Consider the following Interval family I,

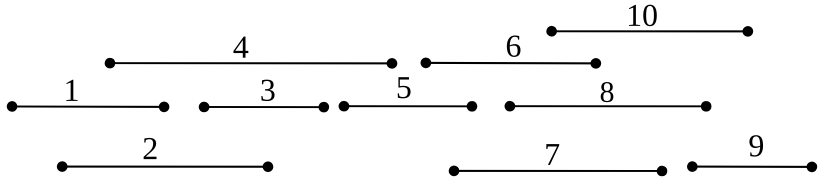

Fig 1: Interval family I

The corresponding interval graph G is as follows,

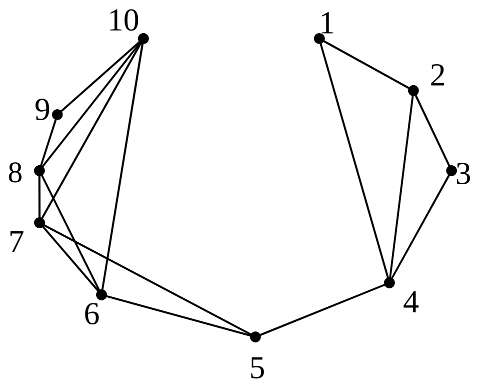

Fig 2: Interval Graph G

Restrained dominating set RDS of the interval graph is RDS = {4, 8}

<V - RDS> = {1, 2, 3, 5, 6, 7, 9, 10}

The Vertex induced sub graph < V - RDS> is as follows,

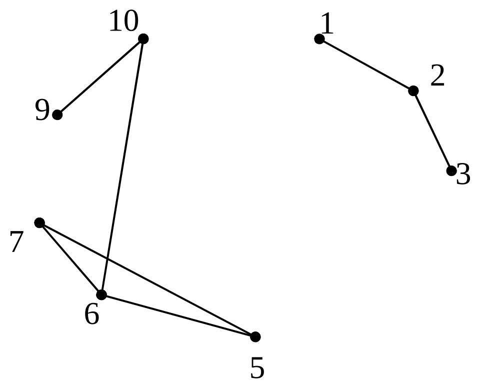

Fig 3: Vertex Induced Sub Graph < V - RDS>

The corresponding neighbourhoods of each vertex are as follows,

Nbd (1) = { 1, 2 }

Nbd (2) = { 1, 2, 3 }

Nbd (3) = { 2, 3 }

Nbd (5) = { 5, 6, 7 }

Nbd (6) = { 5, 6, 7, 10 }

Nbd (7) = { 5, 6, 7 }

Nbd (9) = { 9, 10 }

Nbd (10) = { 6, 9, 10 }

To find the minimum 2- tuple dominating set, we have to compute all the p – th numbered adjacent vertices.

$M_i(v)\backslash v$	1	2	3	5	6	7	9	10
$M_0(v)$	1	1	2	5	5	5	9	6
$M_1(v)$	2	2	3	6	6	6	10	9
$M_2(v)$	-	3	-	7	7	7	-	10
$M_3(v)$	-	-	-	-	10	-	-	-

First we set f (j) = 0, $\forall j \in V$.

In Step 2, set i = 1, D_1 = ϕ, that is initially D_1 is empty. Step 2 repeats for n_1 times. Here n_1=8, the number of vertices in the induced sub graph <V-RDS>.

Now we will illustrate the iterations in the following way. First set f (j) = 0, $\forall j \in V$.

Set f(1) = 0, f(2) = 0.

Iteration 1:

For the first iteration i =1

N [1] = {1, 2}

W_1(f) = f(N[1])

W_1(f) = f(1) + f(2) = 0

The first condition of if end if is satisfied. Since W_1(f) = 0, we find M_0(1) = 1, M_1(1) = 2. Then set f(1) = 1, f(2) = 1.

Also set D_1 = $\phi \cup$ {1, 2}

$\Rightarrow$ D_1 = {1, 2}

Iteration 2:

For the second iteration i =2

N [2] = {1, 2, 3}

W_2(f) = f(N[2])

W_2(f) = f(1) + f(2) + f(3) = 1+1+0

W_2(f) = 2

In this iteration D_1 could not be calculated. Hence D_1 remains same and i is being increased to 3.

Iteration 3:

For the third iteration i =3

N [3] = {2, 3}

W_3(f) = f(N[3])

W_3(f) = f(2) + f(3) = 1+0

W_3(f) = 1

Hence the domination is not satisfied. The else if condition of if end if is satisfied. Now we check f(M_1(3)) = 1 or not. We see that f(M_1(3)) = f(3) = 0 and hence set f(3) = 1. Update D_1 by

$D_1 \cup$ {3} = {1, 2} $\cup$ {3} = {1, 2, 3}

Iteration 4:

For the fourth iteration i =4

N [5] = {5, 6, 7}

W_5(f) = f(N[5])

W_5(f) = f(5) + f(6) + f(7)

W_5(f) = 0+0+0 = 0

The first condition of if end if is satisfied. Since W_5(f) = 0, we find M_0(5) = 5, M_1(5) = 6. Then set f(5) = 1; f(6) = 1.

Also set D_1 = $D_1 \cup$ {5, 6}

$\Rightarrow$ D_1 = {1, 2, 3} $\cup$ {5, 6}

$\Rightarrow$ D_1 = {1, 2, 3,5, 6}

Iteration 5:

For the fifth iteration i = 5

N [6] = {5, 6, 7, 10}

$W_6(f) = f(N[6])$

$W_6(f) = f(5) + f(6) + f(7) + f(10)$

$W_6(f) = 1+1+0+0 = 2$

In this iteration D_1 could not be calculated. Hence the iteration I is being increased to 6.

Iteration 6:

For the sixth iteration i = 6

N [7] = {5, 6, 7}

$W_7(f) = f(N[7])$

$W_7(f) = f(5) + f(6) + f(7)$

$W_7(f) = 1+1+0 = 2$

In this iteration also D_1 could not be calculated. Hence the iteration i is being increased to 7.

Iteration 7:

For the seventh iteration i = 7

N [9] = {9, 10}

$W_9(f) = f(N[9])$

$W_9(f) = f(9) + f(10)$

$W_9(f) = 0+0 = 0$

The first condition of if end if is satisfied. Since $W_9(f) = 0$, we find $M_0(9) = 9$, $M_1(9) = 10$. Then set f(9) = 1, f(10) = 1.

Thus $D_1 = D_1 \cup \{9, 10\}$

$\Rightarrow D_1 = \{1, 2, 3, 5, 6\} \cup \{9, 10\}$

$\Rightarrow D_1 = \{1, 2, 3, 5, 6, 9, 10\}$

Iteration 8:

For the eighth iteration i = 8

N [10] = { 6, 9,10}

$W_{10}(f) = f(N[10])$

W10(f) = f(6) + f(9) + f(10)

$W_{10}(f) = 1+1+1 = 3$

In this iteration also D_1 could not be calculated.

$\therefore D_1 = \{1, 2, 3, 5, 6, 9, 10\}$

Cardinality of $D_1 = |D_1| = 7$

$= n_1 - 1$

Where n_1 is the number of vertices of G_1 = < V – RDS >.

Hence the theorem proved.

Theorem 2: If I_i and I_j are any two intervals of I such that I_j is contained in I_i and there is exactly one interval $I_k \neq I_i$, $I_k \in$ RDS that intersects I_j, then the induced sub graph < V – RDS > is disconnected, then the cardinality of double dominating set < V – RDS > is $|D_1| = n_1 - 1,$ n_1 is the number of vertices of the induced sub graph G_1 = < V – RDS >.

Proof: Let $I = \{I_1, I_2, I_3, ..., I_m\}$ be the given interval family and G is an Interval graph corresponding to I. Let I_i and I_j be any two intervals of I such that I_j is contained in I_i. By our assumption there is exactly one interval $I_k \neq I_i$, $I_k \in$ RDS that intersects I_j as a contradiction. Let us take there is one more interval $I_l \neq I_i$, $I_l \in$ RDS, that intersects I_j, then we cannot get the split restrained domination.

Illustration: Consider the following Interval family I,

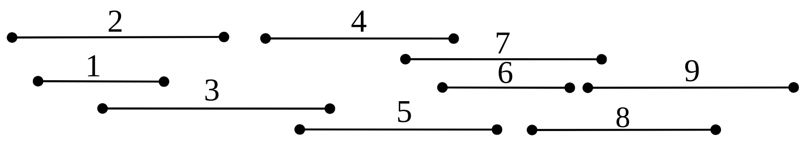

Fig 4: Interval family I

The corresponding interval graph G is as follows,

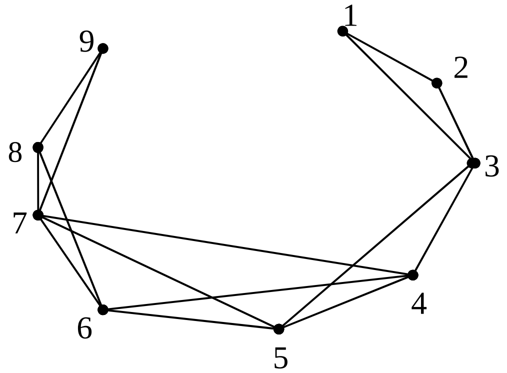

Fig 5: Interval Graph G

Restrained dominating set RDS of the interval graph is RDS = {3, 8} <V - RDS> = {1, 2, 4, 5, 6, 7, 9} The Vertex induced sub graph < V - RDS> is as follows,

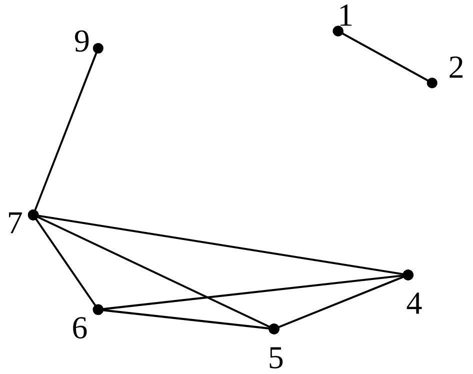

Fig 6: Vertex induced sub graph < V - RDS>

The corresponding neighbourhoods of each vertex are as follows,

Nbd (1) = { 1, 2 }

Nbd (2) = { 1, 2 }

Nbd (4) = { 4, 5, 6, 7 }

Nbd (5) = { 4, 5, 6, 7 }

Nbd (6) = { 4, 5, 6, 7 }

Nbd (7) = { 4, 5, 6, 7, 9 }

Nbd (9) = { 7, 9 }

ISSN: 2321-7782 (Online)

To find the minimum 2- tuple dominating set, we have to compute all the p – th numbered adjacent vertices.

$M_i(v)\backslash v$	1	2	4	5	6	7	9
$M_0(v)$	1	1	4	4	4	4	7
$M_1(v)$	2	2	5	5	5	5	9
$M_2(v)$	-	-	6	6	6	6	-
$M_3(v)$	-	-	7	7	7	7	-

First we set f (j) = 0, $\forall j \in V$.

In Step 2, set i = 1, D_1 = ϕ, that is initially D_1 is empty. Step 2 repeats for n_1 times. Here n_1 = 7, the number of vertices in the induced sub graph G_1 = <V-RDS>.

Now we will illustrate the iterations in the following way. First set f (j) = 0, $\forall j \in V$.

Set f(1) = 0, f(2) = 0.

Iteration 1:

For the first iteration i =1

N [1] = {1, 2}

$W_1(f) = f(N[1])$

$W_1(f) = f(1) + f(2) = 0$

The first condition of if end if is satisfied. Since $W_1(f) = 0$, we find $M_0(1) = 1$, $M_1(1) = 2$. Then set f(1) = 1; f(2) = 1.

Also set $D_1 = \phi \cup \{1, 2\}$

$\Rightarrow D_1 = \{1, 2\}$

Iteration 2:

For the second iteration i =2

N [2] = {1, 2}

$W_2(f) = f(N[2])$

$W_2(f) = f(1) + f(2) = 1+1$

$W_2(f) = 2$

That is the vertex 2 is dominated by the two vertices 1 and 2 of D_1. So in this iteration D_1 could not be calculated. Hence D_1 remains same and i being increased to 3

Iteration 3:

For the third iteration i =3

N [4] = {4, 5, 6, 7}

$W_4(f) = f(N[4])$

$W_4(f) = f(4) + f(5) + f(6) + f(7) = 0+0+ 0+0$

$W_4(f) = 0$

The first condition of if end if is satisfied. Since $W_4(f) = 0$, we find $M_0(4) = 4$,

$M_1(4) = 5$. Then set f(4) = 1; f(5) = 1.

Also set $D_1 = D_1 \cup \{4, 5\}$

$\Rightarrow D_1 = \{1, 2\} \cup \{4, 5\}$

$\Rightarrow D_1 = \{1, 2, 4, 5\}$

Iteration 4:

For the fourth iteration i =4

N [5] = {4, 5, 6, 7}

$W_5(f) = f(N[5])$

$W_5(f) = f(4) + f(5) + f(6) + f(7)$

$W_5(f) = 1+1+0+0 = 2$

In this iteration D_1 remains same and i is being increased to 5.

Iteration 5:

For the fifth iteration i = 5

N [6] = {4, 5, 6, 7}

$W_6(f) = f(N[6])$

$W_6(f) = f(4) + f(5) + f(6) + f(7)$

$W_6(f) = 1+1+0+0 = 2$

In this iteration also D_1 could not be calculated. Hence the iteration i is being increased to 6.

Iteration 6:

For the sixth iteration i = 6

N [7] = {4, 5, 6, 7, 9}

$W_7(f) = f(N[7])$

$W_7(f) = f(4) + f(5) + f(6) + f(7) + f(9)$

$W_7(f) = 1+1+0+0+0 = 2$

In this iteration also D_1 could not be calculated. Hence the iteration i is being increased to 7.

Iteration 7:

For the seventh iteration i = 7

N [9] = {7, 9}

$W_9(f) = f(N[9])$

$W_9(f) = f(7) + f(9)$

$W_9(f) = 0+0 = 0$

The first condition of if end if is satisfied. Since $W_9(f) = 0$, we find $M_0(9) = 7$, $M_1(9) = 9$. Then set $f(7) = 1$, $f(9) = 1$.

Thus $D_1 = D_1 \cup \{7, 9\}$

$\Rightarrow D_1 = \{1, 2, 4, 5\} \cup \{7, 9\}$

$\Rightarrow D_1 = \{1, 2, 4, 5, 7, 9\}$

$\therefore D_1 = \{1, 2, 4, 5, 7, 9\}$

Cardinality of $D_1 = |D_1| = 6$

$= n_1 - 1$

Where n_1 is the number of vertices of $G_1 = < V - RDS >$.

Hence the theorem proved.

Theorem 3: If i, j, k are any three consecutive intervals such that i<j<k and if $j \in$ RDS and i intersects j, j intersects k and i and k does not intersects, then split restrained domination occurs in G and also the cardinality of the 2 – tuple dominating set of disconnected induced sub graph $G_1 = < V - RDS >$ is $|D_1| = n_1 - 1,$ n_1 is the number of vertices of the induced sub graph $G_1 = < V - RDS >$.

Proof: Let $I = \{I_1, I_2, I_3, ..., I_m\}$ be the given interval family and G is an Interval graph corresponding to I. Let i, j, k be three consecutive intervals satisfies the hypothesis, $j \in$ RDS and i intersects j and j intersects k but k does not intersects i, this is true for every interval in the interval family $I = \{I_1, I_2, I_3, ..., I_m\}.$ Let us assume that k intersects I, then we cannot get the split restrained domination, which is shown in the following illustration clearly. So our assumption that k intersects I is wrong, so that the cardinality of the 2 – tuple dominating set is $n_1 - 1$, n_1 is the number of vertices of the induced sub graph $G_1 = < V - RDS >$.

i.e. , $|D_1| = n_1 - 1$

Illustration: Consider the following Interval family I,

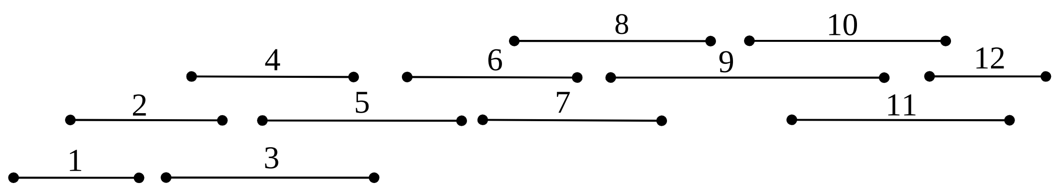

Fig 7: Interval family

The corresponding interval graph G is as follows,

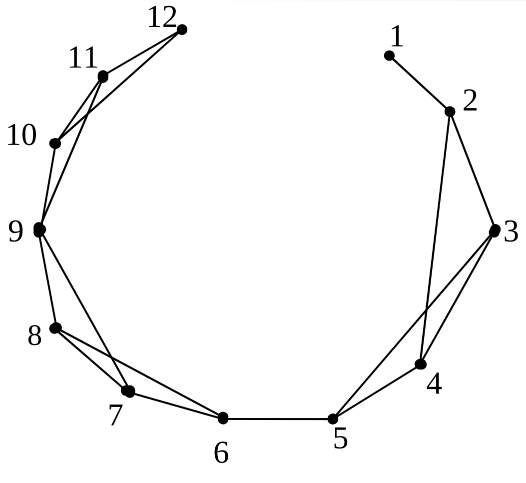

Fig 8: Interval Graph G

Restrained dominating set RDS of the interval graph is RDS = {2, 6, 10} <V - RDS> = {1, 3, 4, 5, 7, 8, 9, 11, 12} The Vertex induced sub graph < V – RDS > is as follows,

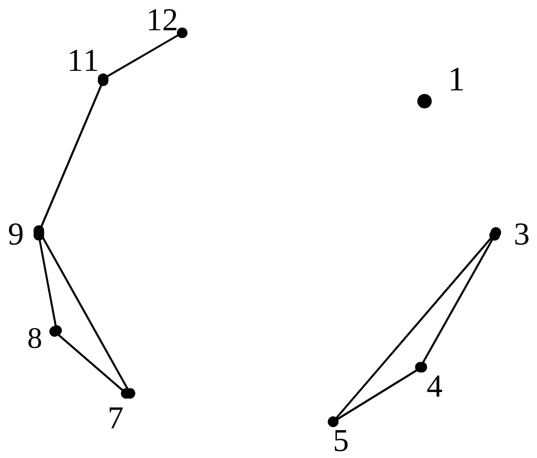

Fig 9: Vertex induced sub graph < V – RDS >

The corresponding neighbourhoods of each vertex are as follows,

Nbd (1) = { 1}

Nbd (3) = { 3, 4, 5 }

Nbd (4) = { 3, 4, 5 }

Nbd (5) = { 3, 4, 5 }

Nbd (7) = { 7, 8, 9 }

Nbd (8) = { 7, 8, 9 }

Nbd (9) = { 7, 8, 9, 11 }

Nbd (11) = { 9, 11, 12 }

Nbd (12) = { 11, 12 }

To find the minimum 2- tuple dominating set, we have to compute all the p – th numbered adjacent vertices.

$M_i(v)\backslash v$	1	3	4	5	7	8	9	11	12
$M_0(v)$	1	3	3	3	7	7	7	9	11
$M_1(v)$	-	4	4	4	8	8	8	11	12
$M_2(v)$	-	5	5	5	9	9	9	12	-
$M_3(v)$	-	-	-	-	-	-	11	-	-

First we set f (j) = 0, $\forall j \in V$.

In Step 2, set i = 1, $D_1 = \phi$, that is initially D_1 is empty. Step 2 repeats for n_1 times. Here n_1 = 9, the number of vertices in the induced sub graph G_1 = < V – RDS >.

Now we will illustrate the iterations in the following way. First set f (j) = 0, $\forall j \in V$. Set f(1) = 0.

Iteration 1:

For the first iteration i =1

N [1] = {1}

$W_1(f) = f(N[1])$

$W_1(f) = f(1) = 0$

The first condition of if end if is satisfied. Since $W_1(f) = 0$, we find $M_0(1) = 1$. Then set f(1) = 1.

Also set $D_1 = \phi \cup \{1\}$

$\Rightarrow D_1 = \{1\}$

Iteration 2:

For the second iteration i =2

N [3] = {3, 4, 5}

$W_2(f) = f(N[3])$

$W_2(f) = f(3) + f(4) + f(5) = 0 + 0 + 0$

$W_2(f) = 0$

Here also the the first condition of if end if is satisfied. Since $W_3 (f) = 0$, we find $M_0(3) = 3$, $M_1(3) = 4$

And set f (3) = 1, f (4) = 1 also set

$\Rightarrow D_1 = \{1\} \cup \{3, 4\}$

$= \{1, 3, 4\}$

Iteration 3:

For the third iteration i =3

N [4] = {3, 4, 5}

$W_3(f) = f(N[4])$

$W_3(f) = f(3) + f(4) + f(5) = 1+1+ 0$

$W_3(f) = 2$

In this iteration D_1 could not be calculated. Hence D_1 remains same and i is being increased to 4.

Iteration 4:

For the fourth iteration i =4

N [5] = {3, 4, 5}

$W_4(f) = f(N[5])$

$W_4(f) = f(3) + f(4) + f(5)$

$W_4(f) = 1+1+0 = 2$

In this iteration also D_1 remains same and i is being increased to 5.

Iteration 5:

For the fifth iteration i = 5

N [7] = {7, 8, 9}

$W_5(f) = f(N[7])$

$W_5(f) = f(7) + f(8) + f(9)$

$W_5(f) = 0+0+0 = 0$

The first condition of if end if is satisfied. Since $W_5(f) = 0$, we find $M_0(7) = 7$, $M_1(7) = 8$. Also set f (7) = 1, f (8) =1.

$D_1 = \{1, 3, 4\} \cup \{7, 8\}$

 ISSN: 2321-7782 (Online)

$\Rightarrow D_1 = \{1, 3, 4, 7, 8\}$

Iteration 6:

For the sixth iteration i = 6

N [8] = {7, 8, 9}

$W_6(f) = f(N[8])$

$W_6(f) = f(7) + f(8) + f(9)$

$W_6(f) = 1+1+0= 2$

In this iteration also D_1 could not be calculated. Hence the iteration i is being increased to 7.

Iteration 7:

For the seventh iteration i = 7

N [9] = {7, 8, 9}

$W_7(f) = f(N[9])$

$W_7(f) = f(7) + f(8) + f(9)$

$W_7(f) = 1+1+0 = 2$

In this iteration also D_1 could not be calculated. Hence the iteration i is being increased to 8.

Iteration 8:

For the eighth iteration i = 8

N [11] = {9, 11, 12}

$W_8(f) = f(N[11])$

$W_8(f) = f(9) + f(11) + f(12)$

$W_8(f) = 0+0+0 = 0$

The first condition of if end if is satisfied. Since $W_8(f) = 0$, we find $M_0(11) = 9$, $M_1(11) = 11$. Also set f (9) = 1, f (11) =1.

$D_1 = \{1, 3, 4, 7, 8\} \cup \{9, 11\}$

$\Rightarrow D_1 = \{1, 3, 4, 7, 8, 9, 11\}$

Iteration 9:

For the ninth iteration i = 9

N [12] = {11, 12}

$W_9(f) = f(N[12])$

$W_9(f) = f(11) + f(12)$

$W_9(f) = 1+0 = 1$

Hence the domination criteria are not satisfied. The else if condition of if end if is satisfied. Now we check f $(M_1(12))$ =1 or not.

We see that f $(M_1(12))$ = f (12) = 0 and hence set f (12) = 1 , update D_1 by

$D_1 = D_1 \cup \{12\}$

= {1, 3, 4, 7, 8, 9, 11, 12}

$\therefore D_1 = \{1, 3, 4, 7, 8, 9, 11, 12\}$

Cardinality of $D_1 = |D_1| = 8$

$= n_1 - 1$

Where n_1 is the number of vertices of $G_1 = < V - RDS >$.

Hence the theorem proved.

II. Conclusion

Resolving The

To find A 2- Tuple dominating set of an induced sub graph of a split restrained dominating set of an interval graph using an algorithm of some special classes of interval graphs has been the main focus of the paper. Especially, the nature of the intervals played a major role in determining the 2- Tuple dominating set of an induced sub graph of a split restrained dominating set of the interval graphs with amazing ease. Some categorized graphs have been chosen in the process of exploration. In future, efforts will be put to identify the interval graphs with split restrained dominating sets.

Acknowledgement

The Author is grateful to the referees for their voluble comments which have lead to improvements in the presentation of the paper. This research was supported impart by the DST-SERB Project, New Delhi, and S.V.University, Tirupati, India

References

1. Dr.A.Sudhakaraiah, E.Gnana Deepika, V.Rama Latha, To find a 2-tuple dominating set of an induced subgraph of a non-split dominating set of an interval graph using an algorithm, International Journal of Engineering Research and Technology, ISSN:2278-0181, Vol. 1 Issue 3, May-2012.
2. G.S.Domke, J.H. Hattingh, M.A.Henning, and L.R.Markus, Restrained domination in graphs with minimum degree two. J.Combin.Math.Combin.Comput.35 (2000) 239-254.
3. Tarasankar Pramanik, Sukumar Mondal and Madhumangal Pal, Minimum 2-tuple dominating set of an interval graph.
4. F. Harary and T.W.Haynes, Double domination in graphs, Ars Combination, 55 (2000) 201-213.
5. E. Sampath Kumar, P.S. Neeralagi, The neighbourhood of graph, Indian J.Pure.Appl.Math., 16,1985, 126-132.

Author Profile

Dr. A.Sudhakaraiah, M.Sc., Ph.D., Assistant Professor in Mathematics in the Department of Future Studies, Sri Venkateswara University, Tirupati. He has 14 years teaching experience in P.G, 3 Ph.D. Degrees awarded, 2 Ph.D degrees submitted, 10 M.Phil degrees awarded, 40 papers published in international journals, 40 papers presented national and international seminars, 3 International Memberships and also IAO.

Mr. K.Ramakrishna doing Ph.D. in Department of Mathematics S.V.University Tirupati. He has 2 years teaching experience and also One paper is published in International journal.

Dr.E.Gnanadeepika M.Sc., Ph.D., in Dept. of Mathematics, S.V.University Tirupati and have 20 publications and 9 seminars. At present working as a Lecturer in Govt. Polytechnic college, Tirupati-517502.