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Almost vg-Open Mappings

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Abstract: The aim of this paper is to introduce and study the concept of almost vg-open mappings and the interrelationship between other almost open maps..

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1. Introduction

Mappings play an important role in the study of modern mathematics, especially in Topology and Functional analysis. Open mappings are one such mapping which is studied for different types of open sets by various mathematicians for the past many years. Author of the present paper studied *vg*-open mappings in the year 2011. In the present paper author tried to study a new variety of open map called almost *vg*-open map.

§2. PRELIMINARIES:

Definition 2.1: A$\subseteq$ X is said to be

a) regular open[pre-open; semi-open; α-open; β-open] if A= int(cl(A)) [A $\subseteq$int(cl(A)); A$\subseteq$ cl(int(A)); A$\subseteq$ int(cl(int(A))); A$\subseteq$ cl(int(cl(A)))] and regular closed[pre-closed; semi-closed; α-closed; β-closed] if A = cl(int(A))[cl(int(A)) $\subseteq$ A; int(cl(A))$\subseteq$ A; cl(int(cl(A)))$\subseteq$ A; int(cl(int(A)))$\subseteq$ A]

b) *v*-open if there exists a r-open set U such that U $\subseteq$ A $\subseteq$ cl(U).

c) g-closed[rg-closed] if cl(A)$\subset$U[rcl(A)$\subset$U] whenever A$\subset$U and U is open[r-open] in X.

d) sg-closed[gs-closed] if scl(A)$\subset$U whenever A$\subset$U and U is s-open[open] in X.

e) pg-closed[gp-closed] if pcl(A)$\subset$U whenever A$\subset$U and U is p-open[open] in X.

f) αg-closed[gα-closed; rgα-closed] if αcl(A)$\subset$U whenever A$\subset$U and U is α-open[open; rα-open] in X.

g) βg-closed[gβ-closed] if βcl(A)$\subset$U whenever A$\subset$U and U is β-open[open] in X.

h) *v*g-closed if *vcl*(A)$\subset$U whenever A$\subset$U and U is *v*-open in X.

i) g-open[rg-open; sg-open; gs-open; pg-open; gp-open; *vg*-open; αg-open; gα-open; rgα-open; βg-open; gβ-open] if its complement X - A is g-closed[rg-closed; sg-closed; gs-closed; pg-closed; gp-closed; *vg*-closed; αg-closed; gα-closed; rgα-closed; βg-closed; gβ-closed].

Remark 1: We have the following implication diagrams for open sets.

g.open gs.open
↓ ↓
rαg.open → rgα.open → rg.open → ***vg*.open** ← sg.open ← βg.open
↑ ↑ ↑ ↑ ↑
→ → rα.open → → *v*.open →
↑ ↓ ↑ ↑
r.open → π.open → open → α.open → s.open → β.open
↓ ↓ ↓
πg.open p.open → ω.open gα.open
↓ ↓
gp.open ← pg.open rω.open

Definition 2.2: A function *f:X→Y* is said to be

a) continuous [resp: semi-continuous, r-continuous, *v*-continuous] if the inverse image of every open set is open [resp: semi open, regular open, *v*-open].

b) irresolute [resp: r-irresolute, *v*-irresolute] if the inverse image of every semi open [resp: regular open, *v*-open] set is semi open [resp: regular open, *v*-open].

c) open[resp: r-open, semi-open, pre-open, α-open, β-open, rα-open] if the image of every open set is open[resp: regular-open, semi-open, pre-open, α-open, β-open, rα-open].

d) g-continuous [resp: rg-continuous] if the inverse image of every closed set is g-closed [resp: rg-closed].

e) g-open[resp: rg-open, sg-open, pg-open, αg-open, βg-open, rαg-open, rgα-open, gs-open, gp-open, gα-open] if the image of every open set is g-open[resp: rg-open, sg-open, pg-open, αg-open, βg-open, rαg-open, rgα-open, gs-open, gp-open, gα-open].

Definition 2.3: *X* is said to be *v*-regular space (or *v*-T_3 space) if for a open set F and a point x∉F, there exists disjoint *v*-open sets G and H such that F ⊆ G and x∈H.

Definition 2.4: *X* is said to be $T_{1/2}$[r-$T_{1/2}$] if every (regular) generalized closed set is (regular) closed.

2. ALMOST *VG*-OPEN MAPPINGS

Definition 3.1: A function *f: X→Y* is said to be almost *vg*-open if the image of every r-open set in *X* is *vg*-open in *Y*.

Theorem 3.1: Every *vg*-open map is almost *vg*-open but not conversely.

Note 1: We have the following implication diagrams for almost open mappings.

(i) al.g.open al.gs.open
↓ ↓
al.rαg.open → al.rgα.open → al.rg.open → al.***vg*.open** ← al.sg.open ← al.βg.open
↑ ↑ ↑ ↑ ↑
→ → al.rα.open → → al.*v*.open →
↑ ↓ ↑ ↑
al.r.open → al.π.open → al.open → al.α.open → al.s.open → al.β.open
↓ ↓ ↓
al.πg.open al.p.open → al.ω.open al.gα.open
↓ ↓
al.gp.open ← al.pg.open al.rω.open None is reversible

(ii)

g.open gs.open
↓ ↓
rαg.open → rgα.open → rg.open → al.***vg*.open** ← sg.open ← βg.open
↑ ↑ ↑ ↑ ↑
→ → rα.open → → *v*.open →
↑ ↓ ↑ ↑
r.open → π.open → open → α.open → s.open → β.open None is reversible

(iii) If *v*GO(Y) = RO(Y), then the reverse relations hold for all almost open maps.

al.g.open al.gs.open
↑↓ ↑↓
al.rαg.open ↔ al.rgα.open ↔ al.rg.open ↔ al.***vg*.open** ↔ al.sg.open ↔ al.βg.open
↑↓ ↑↓ ↑↓ ↑↓ ↑↓
↔ ↔ al.rα.open ↔ ↔ ↔ al.*v*.open ↔
↑↓ ↑↓ ↑↓ ↑↓
al.r.open ↔ al.π.open ↔ al.open ↔ al.α.open ↔ al.s.open ↔ al.β.open

Example 1: Let $X = Y = \{a, b, c\}$; $\tau = \{\varphi, \{a\}, \{a, b\}, X\}$; $\sigma = \{\varphi, \{a, c\}, Y\}$. Let $f:X\to Y$ be defined $f(a) = c$, $f(b) = b$ and $f(c) = a$. Then f is almost *vg*-open.and almost *v*-open but not *v*-open.

Example 2: Let $X = Y = \{a, b, c\}$; $\tau = \{\varphi, \{a\}, \{b, c\}, X\}$; $\sigma = \{\varphi, \{a, c\}, Y\}$. Let $f:X\to Y$ be defined $f(a) = b$, $f(b) = c$ and $f(c) = a$. Then f is almost *vg*-open but not almost *v*-open.

Theorem 3.2:

(i) If (Y, σ) is discrete, then f is almost open of all types.

(ii) If f is almost open[almost *r*-open] and g is *vg*-open then gof is almost *vg*-open.

(iii) If f and g are almost *r*-open then gof is almost *vg*-open.

Corollary 3.1: If f is almost open[almost r-open] and g is *g*-[*rg-; sg-; gs-; βg-; rαg-; rgα-*; *rα-*; *α-; s-; p-*; *β-; v-;* π-; r-] open then gof is almost *vg*-open.

Corollary 3.2: If f is open[r-open] and g is *g*-[*rg-; sg-; gs-; βg-; rαg-; rgα-*; *rα-*; *α-; s-; p-*; *β-;*π-] open then gof is almost *vg*-open.

Theorem 3.3: If $f: X\to Y$ is almost *vg*-open, then $f(A^o)\subset vg(f(A))^o$

Proof: Let $A\subseteq X$ be r-open and $f: X\to Y$ is almost *vg*-open gives $f(A^o)$ is *vg*-open in Y and $f(A^o)\subset f(A)$ which in turn gives $vg(f(A^o))^o\subset vg(f(A))^o$ - - - (1)

Since $f(A^o)$ is *vg*-open in Y, $vg(f(A^o))^o = f(A^o)$ - - - - - - - - - - -(2)

combining (1) and (2) we have $f(A^o)\subset vg(f(A))^o$ for every subset A of X.

Remark 2: Converse is not true in general.

Corollary 3.3: If $f: X\to Y$ is *g*-[*rg-; sg-; gs-; βg-; rαg-; rgα-*; *rα-*; *α-; s-; p-*; *β-; v-;* π-; r-] open, then $f(A^o)\subset vg(f(A))^o$

Proof: For $A\subset X$ is r-open and $f:X\to Y$ is *g*-open, we get $f(A^o)\subset g(f(A))^o$ and so we get $f(A^o)\subset vg(f(A))^o$. [since *g*-open set is *vg*-open, $g(f(A))^o = vg(f(A))^o$].

Similarly we can prove the remaining results.

Corollary 3.4: If $f: X\to Y$ is al-*g*-[al-*rg-;* al-*sg-;* al-*gs-;* al-*βg-;* al-*rαg-;* al-*rgα-*; al-*r-*; al-*rα-*; al-*α-;* al-*s-;* al-*p-*; al-*β-;* al-*v-;* al-π-] open, then $f(A^o)\subset vg(f(A))^o$

Theorem 3.4: If f:$X\to Y$ is almost vg-open and A$\subseteq X$ is r-open, f(A) is τ_{vg}-open in Y.

Proof: Let A$\subset$X be r-open and f:X$\to$Y is almost vg-open $\Rightarrow f(A^o)\subset vg(f(A))^o \Rightarrow f(A) \subset vg(f(A))^o$, since $f(A) = f(A^o)$. But $vg(f(A))^o\subset f(A)$. Combining we get $f(A) = vg(f(A))^o$. Hence f(A) is τ_{vg}-open in Y.

Corollary 3.5: If f:$X\to Y$ is al-g-[al-rg-; al-sg-; al-gs-; al-βg-; al-$r\alpha g$-; al-$rg\alpha$-; al-r-; al-$r\alpha$-; al-α-; al-s-; al-p-; al-β-; al-v-; al-π-]open, then f(A) is τ_{vg} open in Y if A is r-open set in X.

Theorem 3.5: If f:$X\to Y$ is g-[rg-; sg-; gs-; βg-; $r\alpha g$-; $rg\alpha$-; $r\alpha$-; α-; s-; p-; β-; v-; π-; r-]open and A$\subseteq X$ is r-open, f(A) is τ_{vg}-open in Y.

Proof: For A$\subset$X is r-open and f:X$\to$Y is rg-open, f(A) is τ_{rg}-open in Y and so f(A) is τ_{vg}-open in Y. [since g-open set is vg-open]. Similarly we can preove the remaining results.

Theorem 3.6: If $vg(A)^o = r(A)^o$ for every A$\subset$ Y, then the following are equivalent:

a) f: X$\to$ Y is almost vg-open map

b) $f(A^o)\subset vg(f(A))^o$

Proof: (a) $\Rightarrow$(b) follows from theorem 3.3.

(b) $\Rightarrow$ (a) Let A be any r-open set in X, then $f(A) = f(A^o)\subset vg(f(A))^o$ by hypothesis. We have $f(A) \subset vg(f(A))^o$, which implies f(A) is vg-open. Therefore f is almost vg-open.

Theorem 3.7: If $v(A)^o = r(A)^o$ for every A$\subset$ Y, then the following are equivalent:

a) f: X$\to$ Y is vg-open map

b) $f(A^o)\subset vg(f(A))^o$

Proof: (a) $\Rightarrow$(b) follows from theorem 3.3.

(b) $\Rightarrow$ (a) Let A be any r-open set in X, then $f(A) = f(A^o)\subset vg(f(A))^o$ by hypothesis. We have $f(A) \subset vg(f(A))^o$, which implies f(A) is vg-open. Therefore f is almost vg-open.

Theorem 3.8: f:$X\to Y$ is almost vg-open iff for each subset S of Y and each U$\in$RO(X, f^{-1}(S)), there is an vg-open set V of Y such that S$\subseteq$V and f^1(V) $\subseteq$ U.

Proof: Let S$\subseteq Y$ and U$\in$RO(X, f^1(S)). Then V = f(U) is vg-open in Y as f is almost vg-open. $f^{-1}(S)\subseteq U \Rightarrow S\subseteq f(U) = V$ and $f^{-1}(V) = f^{-1}(f(U)) = U$

Conversely Let U be r-open in X. Then by hypothesis there exists an vg-open set V of Y, such that $f^{-1}(V) \subseteq U$ and so $V \subseteq f(U)$. Thus f(U) is vg-open in Y. Therefore f is almost vg-open.

Remark 3: Composition of two almost vg-open maps is not almost vg-open in general.

Theorem 3.9: Let X, Y, Z be topological spaces and every vg-open set is r-open in Y. Then the composition of two almost vg-open maps is almost vg-open.

Proof: (a) Let f and g be almost vg-open maps. Let A be any r-open set in $X \Rightarrow f$(A) is r-open in Y (by assumption) $\Rightarrow g(f(A)) = g\ o\ f(A)$ is vg-open in Z. Therefore gof is almost vg-open.

Theorem 3.10: Let X, Y, Z be topological spaces and every v-open set is open [r-open] in Y. Then the composition of two v-open[r-open] maps is almost vg-open.

Proof: (a) Let *f*, g be *v*-open maps. Let A be *r*-open in *X* ⇒ *f*(A) is *v*-open and so open in *Y* (by assumption) ⇒ *g*(*f* (A)) = *gof*(A) is *v*-open in *Z*. Hence *gof* is almost *vg*-open [since every *v*-open set is *vg*-open].

Theorem 3.11: Let *X*, *Y*, *Z* be topological spaces and every *g*-[*rg*-*; sg*-*; gs*-*; βg*-*; rαg*-*; rgα*-; *rα*-; *α*-*; s*-*; p*-; *β*-*;* π-]open set is *r*-open in *Y*. Then the composition of two al-*g*-[al-*rg*-*;* al-*sg*-*;* al-*gs*-*;* al-*βg*-*;* al-*rαg*-*;* al-*rgα*-; al-*r*-; al-*rα*-; al-*α*-*;* al-*s*-*;* al-*p*-; al-*β*-*;* al-*v*-*;* al-π-]open maps is almost *vg*-open.

Proof: Let A be *r*-open set in X, then *f*(A) is *sg*-open in Y and so *r*-open in *Y* (by assumption) ⇒ *g*(*f* (A)) = *gof*(A) is *sg*-open in *Z*. Hence *gof* is almost *vg*-open [since every *sg*-open set is *vg*-open].

Corollary 3.6: Let *X*, *Y*, *Z* be topological spaces and every *g*-[*rg*-*; sg*-*; gs*-*; βg*-*; rαg*-*; rgα*-; *rα*-; *α*-*; s*-*; p*-; *β*-*;* π-]open set is open [*r*-open] in *Y*. Then the composition of two *g*-[*rg*-*; sg*-*; gs*-*; βg*-*; rαg*-*; rgα*-; *rα*-; *α*-*; s*-*; p*-; *β*-*; v*-*;* π-; r-]open maps is almost *vg*-open.

Proof: Let A be *r*-open set in X, then *f*(A) is *sg*-open in Y and so open in *Y* (by assumption) ⇒ *g*(*f* (A)) = *gof*(A) is *sg*-open in *Z*. Hence *gof* is almost *vg*-open [since every *sg*-open set is *vg*-open].

Example 3: Let *X* = *Y* = Z = {a, b, c}; τ = {φ, {a}, {a, b}, *X*}; σ = {φ, {a, c}, *Y*} and η = {φ, {a}, {b, c}, *Z*}. *f*:*X*→*Y* be defined *f*(a) = c, *f*(b) = b and *f*(c) = a and *g*:*Y*→*Z* be defined *g*(a) = b, *g*(b) = a and *g*(c) = c, then *g*, *f* and *g o f* are almost *vg*-open.

Theorem 3.12: If *f*:*X*→*Y* is almost g-open[almost rg-open], g:*Y*→*Z* is *vg*-open and *Y* is $T_{½}$ [*r*-$T_{½}$] then *gof* is almost *vg*-open.

Proof: (a) Let A be r-open in *X*. Then *f*(A) is *g*-open and so open in *Y* as *Y* is $T_{½}$ ⇒ *g*(*f* (A)) = *gof* (A) is *vg*-open in *Z* (since *g* is *vg*-open). Hence *gof* is almost *vg*-open.

Theorem 3.13: If *f*:*X*→*Y* is *g*-open[*rg*-open], *g*:*Y*→*Z* is *g*-[*rg*-*; sg*-*; gs*-*; βg*-*; rαg*-*; rgα*-; *rα*-; *α*-*; s*-*; p*-; *β*-*;* π-]open and *Y* is $T_{½}$[*r*-$T_{½}$], then *gof* is almost *vg*-open.

Proof: Let A be r-open set in X, then *f*(A) is *g*-open in Y and so open in *Y* (by assumption) ⇒ *g*(*f* (A)) = *gof*(A) is *gs*-open in *Z*. Hence *gof* is almost *vg*-open [since every *gs*-open set is *vg*-open].

Corollary 3.7: If *f*:*X*→*Y* is almost *g*-open[almost *rg*-open], *g*:*Y*→*Z* is *g*-[*rg*-*; sg*-*; gs*-*; βg*-*; rαg*-*; rgα*-; *rα*-; *α*-*; s*-*; p*-; *β*-*; v*-*;* π-; r-]open and *Y* is $T_{½}$ [*r*-$T_{½}$] then *gof* is almost *vg*-open.

Proof: (a) Let A be r-open in *X*. Then *f*(A) is *g*-open and so open in *Y* as *Y* is $T_{½}$ ⇒ *g*(*f* (A)) = *gof* (A) is *v*-open in *Z* (since *g* is *v*-open). Hence *gof* is almost *vg*-open[since every *v*-open set is *vg*-open].

Theorem 3.14: If *f*:*X*→*Y*, *g*:*Y*→*Z* be two mappings such that *gof* is almost *vg*-open [almost *r*-open] then the following statements are true.

a) If *f* is continuous [*r*-continuous] and surjective then *g* is almost *vg*-open.

b) If *f* is g-continuous[resp: rg-continuous], surjective and *X* is $T_{½}$ [resp: *r*-$T_{½}$] then *g* is almost *vg*-open.

Proof: (a) For A r-open in *Y*, f^{1}(A) open in *X* ⇒ (*g o f*)(f^{1}(A)) = g(A) *vg*-open in *Z*. Hence *g* is almost *vg*-open.

Similarly one can prove the remaining parts and hence omitted.

Corollary 3.8: If *f*:*X*→*Y*, g:*Y*→*Z* be two mappings such that *gof* is -[*rg*-*; sg*-*; gs*-*; βg*-*; rαg*-*; rgα*-; *rα*-; *α*-*; s*-*; p*-; *β*-*; v*-*;* π-; r-]open then the following statements are true.

a) If *f* is continuous [*r*-continuous] and surjective then *g* is almost *vg*-open.

b) If *f* is g-continuous[rg-continuous], surjective and *X* is $T_{½}$ [*r*-$T_{½}$] then *g* is almost *vg*-open.

Theorem 3.15: If X is vg-regular, $f:X\rightarrow Y$ is r-open, r-continuous, almost vg-open surjective and $A^o = A$ for every vg-open set in Y then Y is vg-regular.

Proof: Let $p\in U\in vGO(Y)$, $\exists$ a point $x\in X \ni f(x) = p$ by surjection. Since X is vg-regular and f is nearly-continuous, $\exists$ $V\in RC(X)$ $\ni x\in V^o\subset V\subset f^{-1}(U)$ which implies $p\in f(V^o)\subset f(V) \subset U$-- - - - - - - - - (1)

for f is vg-open, $f(V^o)\subset U$ is vg-open. By hypothesis $f(V^o)^o = f(V^o)$ and $f(V^o)^o = \{f(V)\}^o$-- - - - - - -(2)

combaining (1) and (2) $p\in f(V)^o\subset f(V) \subset U$ and $f(V)$ is r-closed. Hence Y is vg-regular.

Corollary 3.9: If X is vg-regular, $f:X\rightarrow Y$ is r-open, r-continuous, almost vg-open, surjective and $A^o = A$ for every r-open set in Y then Y is vg-regular.

Theorem 3.16: If $f:X\rightarrow Y$ is almost vg-open and $A\in RO(X)$, then $f_A:(X,\tau(A))\rightarrow(Y,\sigma)$ is almost vg-open.

Proof: Let F be an r-open set in A. Then $F = A\cap E$ for some r-open set E of X and so F is r-open in $X \Rightarrow f(A)$ is vg-open in Y. But $f(F) = f_A(F)$. Therefore f_A is almost vg-open.

Theorem 3.17: If $f:X\rightarrow Y$ is almost vg-open, X is $rT_{½}$ and A is rg-open set of X then $f_A:(X,\tau(A))\rightarrow(Y,\sigma)$ is almost vg-open.

Proof: Let F be a r-open set in A. Then $F = A\cap E$ for some r-open set E of X and so F is r-open in $X \Rightarrow f(A)$ is vg-open in Y. But $f(F) = f_A(F)$. Therefore f_A is almost vg-open.

Corollary 3.10: If $f:X\rightarrow Y$ is g-[rg-; sg-; gs-; βg-; $r\alpha g$-; $rg\alpha$-; r-; $r\alpha$-; α-; s-; p-; β-; v-; π-] open and $A\in RO(X)$, then $f_A:(X,\tau(A))\rightarrow(Y,\sigma)$ is almost vg-open.

Proof: Let F be an r-open set in A. Then $F = A\cap E$ for some r-open set E of X and so F is r-open in $X \Rightarrow f(A)$ is $r\alpha g$-open in Y. But $f(F) = f_A(F)$. Therefore f_A is almost vg-open[since every $r\alpha g$-open set is vg-open].

Corollary 3.11: If $f:X\rightarrow Y$ is al-g-[al-rg-; al-sg-; al-gs-; al-βg-; al-$r\alpha g$-; al-$rg\alpha$-; al-r-; al-$r\alpha$-; al-α-; al-s-; al-p-; al-β-; al-v-; al-π-] open and $A\in RO(X)$, then $f_A:(X,\tau(A))\rightarrow(Y,\sigma)$ is almost vg-open.

Proof: Let F be an r-open set in A. Then $F = A\cap E$ for some r-open set E of X and so F is r-open in $X \Rightarrow f(A)$ is $rg\alpha$-open in Y. But $f(F) = f_A(F)$. Therefore f_A is almost vg-open[since every $rg\alpha$-open set is vg-open].

Theorem 3.18: If $f_i:X_i\rightarrow Y_i$ be almost vg-open for i =1, 2. Let f: $X_1xX_2\rightarrow Y_1xY_2$ be defined as $f(x_1,x_2) = (f_1(x_1), f_2(x_2))$. Then $f:X_1xX_2\rightarrow Y_1xY_2$ is almost vg-open.

Proof: Let $U_1xU_2 \subseteq X_1xX_2$ where U_i is r-open in X_i for i = 1,2. Then $f(U_1xU_2) = f_1(U_1)$ x $f_2(U_2)$ is vg-open set in Y_1xY_2. Hence f is almost vg-open.

Corollary 3.12: If $f_i:X_i\rightarrow Y_i$ be al-g-[al-rg-; al-sg-; al-gs-; al-βg-; al-$r\alpha g$-; al-$rg\alpha$-; al-r-; al-$r\alpha$-; al-α-; al-s-; al-p-; al-β-; al-v-; al-π-]open for i =1, 2.

Let $f:X_1xX_2\rightarrow Y_1xY_2$ be defined as $f(x_1,x_2) = (f_1(x_1), f_2(x_2))$, then $f:X_1xX_2\rightarrow Y_1xY_2$ is almost vg-open.

Proof: Let $U_1xU_2 \subseteq X_1xX_2$ where U_i is r-open in X_i for i = 1,2. Then $f(U_1xU_2) = f_1(U_1)$ x $f_2(U_2)$ is vg-open set in Y_1xY_2 [since every α-open set is vg-open]. Hence f is almost vg-open.

Corollary 3.13: If $f_i:X_i\rightarrow Y_i$ be g-[rg-; sg-; gs-; βg-; $r\alpha g$-; $rg\alpha$-; r-; $r\alpha$-; α-; s-; p-; β-; v-; π-]open for i =1, 2. Let $f:X_1xX_2\rightarrow Y_1xY_2$ be defined as $f(x_1,x_2) = (f_1(x_1), f_2(x_2))$, then $f:X_1xX_2\rightarrow Y_1xY_2$ is almost vg-open.

Proof: Let $U_1xU_2 \subseteq X_1xX_2$ where U_i is r-open in X_i for i = 1,2. Then $f(U_1xU_2) = f_1(U_1)$ x $f_2(U_2)$ is vg-open set in Y_1xY_2 [since every α-open set is vg-open]. Hence f is almost vg-open.

Theorem 3.19: Let $h{:}X \to X_1 x X_2$ be almost *vg*-open. Let $f_i{:}X \to X_i$ be defined as $h(x) = (x_1,x_2)$ and $f_i(x) = x_i$. Then f_i: $X \to X_i$ is almost *vg*-open for i =1, 2.

Proof: Let U_1 be r-open in X_1, then $U_1 x X_2$ is r-open in $X_1 x\ X_2$, and $h(U_1 x\ X_2)$ is *vg*-open in X. But $f_1(U_1) = h(U_1 x\ X_2)$, therefore f_1 is almost *vg*-open. Similarly we can show that f_2 is also almost *vg*-open and thus $f_i{:}X \to X_i$ is almost *vg*-open for i = 1, 2.

Corollary 3.14: Let h: $X \to X_1 x X_2$ be al-*g*-[al-*rg*-; al-*sg*-; al-*gs*-; al-βg-; al-$r\alpha g$-; al-$rg\alpha$-; al-*r*-; al-$r\alpha$-; al-α-; al-*s*-; al-*p*-; al-β-; al-*v*-; al-π-] open.

Let f_i: $X \to X_i$ be defined as $h(x) = (x_1, x_2)$ and $f_i(x) = x_i$. Then f_i: $X \to X_i$ is almost *vg*-open for i =1,2.

Proof: Let $U_1 x U_2 \subseteq X_1 x X_2$ where U_i is *r*-open in X_i for i = 1,2. Then $f(U_1 x U_2) = f_1(U_1)\ x\ f_2(U_2)$ is *vg*-open set in $Y_1 x Y_2$ [since every β-open set is *vg*-open]. Hence *f* is almost *vg*-open.

Corollary 3.15: Let h: $X \to X_1 x X_2$ be *g*-[*rg*-; *sg*-; *gs*-; βg-; $r\alpha g$-; $rg\alpha$-; *r*-; $r\alpha$-; α-; *s*-; *p*-; β-; *v*-; π-] open.

Let f_i: $X \to X_i$ be defined as $h(x) = (x_1, x_2)$ and $f_i(x) = x_i$. Then f_i: $X \to X_i$ is almost *vg*-open for i =1,2.

Proof: Let $U_1 x U_2 \subseteq X_1 x X_2$ where U_i is r-open in X_i for i = 1,2. Then $f(U_1 x U_2) = f_1(U_1)\ x\ f_2(U_2)$ is *vg*-open set in $Y_1 x Y_2$ [since every *s*-open set is *vg*-open]. Hence *f* is almost *vg*-open.

Direct manipulation works best when there is a straight-forward mapping from user actions to visible changes on the screen; typical examples include dragging an object to a new position, stretching it to make it larger, or rotating it around an origin. But some changes have no obvious map-ping (like changing an object's color), and others require more precision than can readily be achieved by direct action (like rotating an object by exact quarter turns or moving it to exactly align with another). One way to permit such operations is to provide commands that carry out the de-sired change, together with a way to invoke the commands as needed. In the context of graphical interfaces, the commands are usually invoked by pressing a button in a tool-bar or selecting a choice from a menu. We call this style of interaction in-direct manipulation, since an action on one object (a button, for instance) causes a change to another object.

3. Conclusion

In this paper Author introduced the concept of almost *vg*-open mappings, studied their basic properties and the interrelationship between other almost open maps.

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