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The Demonstration of Assignment Problem for Undergraduate Engineering Study

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Abstract: The assignment downside is one in all the elemental combinatorial improvement issues within the branch of improvement or research in arithmetic. It consists of finding a most weight matching in an exceedingly weighted bipartite graph. In its most general kind, the matter is as follows: There are variety of agents and variety of tasks. Any agent is allotted to perform any task, acquisition some price which will vary counting on the agent-task assignment. it's needed to perform all tasks by distribution precisely one agent to every task and precisely one task to every agent in such the way that the entire price of the assignment is reduced. If the numbers of agents and tasks are equal and therefore the total price of the assignment for all tasks is capable the add of the prices for every agent (or the add of the prices for every task, that is that the same factor during this case), then {the downside|the matter} is termed the linear assignment problem. Commonly, once speaking of the assignment downside with none extra qualification, then the linear assignment downside is supposed.

Keywords: agents, task, time, cost.

I. Introduction

The assignment problem is one of the fundamental combinatorial optimization problems in the branch of optimization or operations research in mathematics. It consists of finding a maximum weight matching in a weighted bipartite graph.

In its most general form, the problem is as follows: There are a number of agents and a number of tasks. Any agent can be assigned to perform any task, incurring some cost that may vary depending on the agent-task assignment. It is required to perform all tasks by assigning exactly one agent to each task and exactly one task to each agent in such a way that the total cost of the assignment is minimized.

If the numbers of agents and tasks are equal and the total cost of the assignment for all tasks is equal to the sum of the costs for each agent (or the sum of the costs for each task, which is the same thing in this case), then the problem is called the linear assignment problem. Commonly, when speaking of the assignment problem without any additional qualification, then the linear assignment problem is meant.

II. Algorithms and Generalization

The Hungarian algorithm is one of many algorithms that have been devised that solve the linear assignment problem within time bounded by a polynomial expression of the number of agents. The assignment problem is a special case of the transportation problem, which is a special case of the minimum cost flow problem, which in turn is a special case of a linear program. While it is possible to solve any of these problems using the simplex algorithm, each specialization has more efficient algorithms designed to take advantage of its special structure. If the cost function involves quadratic inequalities it is called the quadratic assignment problem.

III. Formal Mathematical Defination

The formal definition of the assignment problem (or linear assignment problem) is Given two sets, A and T, of equal size, together with a weight function C : A × T → R. Find a bijection f : A → T such that the cost function:

$$\sum_{a\in A} C(a, f(a))$$

is minimized.

Usually the weight function is viewed as a square real-valued matrix C, so that the cost function is written down as:

$$\sum_{a\in A} C_{a,f(a)}$$

The problem is "linear" because the cost function to be optimized as well as all the constraints contain only linear terms.

The problem can be expressed as a standard linear program with the objective function

$$\sum_{i\in A}\sum_{j\in T} C(i,j)x_{ij}$$

subject to the constraints

$$\sum_{j\in T} x_{ij} = 1 \text{ for } i \in A,$$

$$\sum_{i\in A} x_{ij} = 1 \text{ for } j \in T,$$

$$x_{ij} \geq 0 \text{ for } i, j \in A, T.$$

The variable x_{ij} represents the assignment of agent i to task j, taking value 1 if the assignment is done and 0 otherwise. This formulation allows also fractional variable values, but there is always an optimal solution where the variables take integer values. This is because the constraint matrix is totally unimodular. The first constraint requires that every agent is assigned to exactly one task, and the second constraint requires that every task is assigned exactly one agent.

IV. Example

A company has 4 machines available for assignment to 4 tasks. Any machine can be assigned to any task, and each task requires processing by one machine. The time required to set up each machine for the processing of each task is given in the table below.

TIME (Hours)				
	Task 1	**Task 2**	**Task 3**	**Task 4**
Machine 1	13	4	7	6
Machine 2	1	11	5	4
Machine 3	6	7	2	8
Machine 4	1	3	5	9

The company wants to minimize the total setup time needed for the processing of all four tasks.

If we think of the setup times as transportation costs and define

$$x_{ij} = \begin{cases} 1 & \text{if machine i is assigned to process task j,} \\ 0 & \text{if machine i is not assigned to process task j,} \end{cases}$$

where i = 1, 2, 3, 4 and j = 1, 2, 3, 4, then it is easily seen that what we have is a balanced transportation problem with 4 sources (representing the machines), 4 sinks (representing the tasks), a single unit of supply from each source (representing the availability of a machine), and a single unit of demand at each sink (representing the processing requirement of a task). This particular class of transportation problems is called the assignment problems. These problems can, of course, be solved by the streamlined Simplex algorithm. This is sketched below.

The transportation tableau for this problem is given below.

Sinks

Sources	**1**	**2**	**3**	**4**	
1	13	4	7	6	**1**
2	1	11	5	4	**1**
3	6	7	2	8	**1**
4	1	3	5	9	**1**
	1	**1**	**1**	**1**	

Using the least-cost method, an initial basic feasible solution can be easily obtained; this is shown in the table below.

Sinks

Sources	**1***	**2***	**3***	**4***	
1	13	4 (1)	7 (0)	6 (0)	**1/ 0**
2*	1	11	5	4 (1)	**1/ 0/**
3*	6	7	2 (1)	8	**1/ 0/**
4*	1 (1)	3 (0)	5	9	**1/ 0/**
	1/ 0/	**1/ 0/**	**1/ 0/**	**1/ 0/**	

These assignments are made in the following order: $x_{41} = 1$, $x_{33} = 1$, $x_{42} = 0$, $x_{12} = 1$, $x_{24} = 1$, $x_{14} = 0$, and $x_{13} = 0$. Notice that a standard feature of any basic feasible solution in an assignment problem is that it is degenerate.

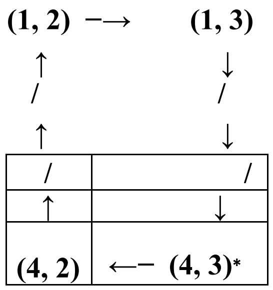
Next, we will use the u-v method to conduct the optimality test. The modifiers associated with the current solution, based on the initial assignment $u_1 = 0$, are shown in the tableau below.

Sinks

Sources	1	2	3	4		Modifier
1	13	4 / 1	7 / 0	6 / 0	1	$u_1 = 0$
2	1	11	5	4 / 1	1	$u_2 = -2$
3	6	7	2 / 1	8	1	$u_3 = -5$
4	1 / 1	3 / 0	5	9	1	$u_4 = -1$
	1	1	1	1		
Modifier	$v_1 = 2$	$v_2 = 4$	$v_3 = 7$	$v_4 = 6$		

It follows that the reduced costs associated with the nonbasic cells are: $\bar{c}_{11} = 11$, $\bar{c}_{21} = 1$, $\bar{c}_{22} = 9$, $\bar{c}_{23} = 0$, $\bar{c}_{31} = 9$, $\bar{c}_{32} = 8$, $\bar{c}_{34} = 7$, $\bar{c}_{43} = -1$, and $\bar{c}_{44} = 4$. Since $\bar{c}_{43} = -1$, the current solution is not optimal, and the entering cell is cell (4, 3).

The stepping-stone path associated with cell (4, 3) is shown below.



Let us choose (arbitrarily) to let x13 exit the current basis. Then, after conducting a pivot according to the above path and updating the modifiers (with the initial assignment u4 = 0), we obtain the new tableau below.

Sinks

Sources	**1**	**2**	**3**	**4**		**Modifier**
1	13	4 / 1	7	6 / 0	**1**	**"1 = 1**
2	1	11	5	4 / 1	**1**	$u_2 = -1$
3	6	7	2 / 1	8	**1**	$u_3 = -3$
4	1 / 1	3 / 0	5 / 0	9	**1**	$u_4 = 0$
	1	**1**	**1**	**1**		
Modifier	$v_1 = 1$	$v_2 = 3$	$v_3 = 5$	$v_4 = 5$		

It is easily seen that the reduced costs associated with the nonbasic cells are: $\bar{c}_{11} = 11$, $\bar{c}_{13} = 1$, $\bar{c}_{21} = 1$, $\bar{c}_{22} = 9$, $\bar{c}_{23} = 1$, $\bar{c}_{31} = 8$, $\bar{c}_{32} = 7$, $\bar{c}_{34} = 6$, and $\bar{c}_{44} = 4$. Since all of these are positive, we conclude that the current solution is (uniquely) optimal. Thus, it is optimal to assign Machine 1 to Task 2, Machine 2 to Task 4, Machine 3 to Task 3, and Machine 4 to Task 1. The total setup time associated with this solution is 11 hours. This completes the solution of the problem.

As noted earlier, every basic feasible solution in an assignment problem is degenerate. Since degeneracy is known to impede progress toward an optimal solution, other algorithms have been developed for the solution of assignment problems.

These other algorithms are designed with the avoidance of large numbers of degenerate pivots in mind; therefore, they are more efficient than the streamlined Simplex algorithm

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