A Novel Approach to Hybrid Genetic Algorithms to Solve Symmetric TSP

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Abstract: The genetic algorithm often meets the occurrence of slow convergence for solving combinatorial optimization. In this study, we present a hybrid genetic algorithm (LSHGA) for symmetric traveling salesman problem. In the algorithm, a modified local search method based on new neighborhood model was contrived as Crossover operation. And a MUT3 operator was introduced as mutation operation. For evading plunge into local extremum, some new ideas are incorporated in our algorithm. The strategy that unites stochastic tournament and elite reservation was used in the process of selection. An idea of reservation ratio was put forward, and the theory of self-adaptive was employed for conforming parameters of LSHGA at the same time. As a result, the population diversity of LSHGA was preserved, and can accomplish convergence quickly. Eight example problems in TSPLIB were showed in the paper to demonstrate the capabilities of the proposed algorithm.

Keywords: Genetic algorithm; TSP; convergence; selection; LSHGA.

I. INTRODUCTION

Traveling Salesman Problem (TSP) is classical combinatorial optimization problem. Some combinatorial optimization problems are NP-hard problems, TSP is one of them. Effective method for solving a NP-hard Problem is that utilize some heuristic algorithms for obtaining approximate result. This paper presents a modified genetic algorithm (LSHGA), which combines Local Search and Genetic Algorithm, for Symmetric TSP. Local Search algorithms are effective heuristics method for many Combinatorial Optimization problems. It is time-consuming for getting the solution if size of issue increases. The choice of the neighborhood structure and the design of the algorithm are a critical issue for contriving a good heuristic algorithm. The neighborhood structure adapt to TSP is k-Opt Shift. The k-Opt Shift is foundation for three distinguished Local heuristics disadvantage of Local Search is easily to run into Search: 2-Opt, 3-Opt and Lin–Kernighan (LK). They determine the neighborhood structure of TSP [1]. And many articles contributed the improvement of LK [2, 3]. However, the critical disadvantage of Local Search is easily to run into local minimum. Combining with other heuristic algorithm, such as Genetic Algorithm, solution for the problem can be obtained. Simulated annealing and Tabu search are effectual means for TSP. Meer [4] improved a simulated annealing algorithm based on Metropolis algorithm Utilizing the concept of genetic algorithm, Ning [5] contrived a Tabu search algorithm which hold crossover operator to solve TSP. Genetic Algorithms is a powerful tool used in combinatorial optimization. Many of the researches have been improving GA by comining it with many other methods [6-9].

II. MODIFIED LOCAL SEARCH HEURISTICS GENETIC ALGORITHM (LSHGA)

The LSHGA which presents in this paper for Symmetric TSP is based on GA frame and asexual reproduction [8]. In the algorithm, a modified local search method based on new neighborhood model was contrived in crossover operation. And a new MUT3 operator was introduced as mutation operation which prevents to get to the local extreme. The process of selection uses the strategy that unites stochastic tournament and elite reservation.
2.1. Encoding and Fitness Function

Natural encoding of a tour and Grefenstence are the two kinds of encoding manner. The natural encoding of a tour is the most natural representation of a TSP tour, as an ordered list of the cities to be visited. Merits of the natural encoding manner are explicitness and concision, and the process of decode is not complex. To avoid the problem posed by permutation representation, some amendatory crossover operator were brought forward: PMX, CX,OX [9]. But these crossover operations have obscure encoding. To keep away from the intricate process of decode, the natural encoding of a tour is used. But these other code manners except the natural encoding of a tour require complicated decodes approach, which consumes large numbers of computer time; especially solve large-scale TSP.

The fitness of a solution is inversely proportion to the length or cost of the solution. i.e., if the length of a tour is greater, the fitness of the tour is less. Equation (1) is introduced for calculating the fitness. Thenumber of cities will be denoted by \( N_{\text{cities}} \). The Fitness Function of LSHGA is:

\[
f(T) = \frac{1}{\sum_{i=1}^{N_{\text{cities}}} (T)} \quad \text{...... (1)},
\]

where \( \Gamma(T) \) is the length of tour \( T \).

2.2. Design of Crossover Operator

To avoid infeasible solution, the scheme of asexual reproduction is betaken, i.e., one parent is chosen into crossover operation and one child is gained. The crossover operator is a modified local search approach whose neighborhood structure is achieved by executing improved Lin-Kernighan. To be more specific, the modified local search is executed on the chosen parent individual, which is the benchmark of the iteration, for achieving a better fitness solution in neighborhood structure. The neighborhood structure, that is, the manner in which the neighborhood is defined, is illustrated in Figure 1. The first solution which is better than the parent is found in the neighborhood as the child of crossover operation. The progress of improved Local Search algorithm is as follows: assume that the parent chromosome represent the tour \( T \).

Step 1: Randomly choose two nodes in parent tour \( T \), they are denoted by \( i \) and \( j \) respectively. Deletes the arc \((i,j)\) which links the two nodes, creating the Hamiltonian path;

Step 2: One of the end points of the Hamiltonian path is fixed (assume node \( i \) is fixed) and stays fixed until the end of the iteration. Choose a node \( k \) whose distance to node \( j \) is least than other nodes. Add the arc \((k,j)\), giving the stem and cycle [9];

Step 3: Delete the arc \((k-1,k)\) and the arc \((k,k+1)\), add the arc \((k,j)\), having a new Hamiltonian path, i.e., a new tour \( T' \);

Step 4: Calculate the fitness of tour \( T' \). If the fitness of tour \( T' \) is better than the one of tour \( T \), then \( T' \) is the child chromosome of \( T \), and the iteration is over, else go to Step5;

Step 5: If the child chromosome is not discovered as exhausting all nodes in the tour \( T \), then the parent tour is regard as the child tour, and the iteration is over, else go to Step1.

2.3. Design of Mutation Operator

A MUT3 operator is employed as the mutation operator in the study [8]. Its process is described as following: Three position numbers are chosen randomly in a chromosome. The chromosome is broken at these positions, so it is divided into 3 parts. The tour pieces are rearranged to create a child of this mutation. For example, suppose a mutation takes place for solution \( T \), denoted by \( ABC \). The lengths of gene pieces are \( p1, p2 \) and \( p3 \), respectively. Let \( C' \) is the inverse permutation of \( C \), i.e \( C' = C_{p3}C_{p2}1\ldots C_{1} \). After mutation course, there is remarkable difference between patent and child solution on genotype.

2.4 Crossover rate, Mutation rate and Reservation ratio
The Crossover rate $P_c$ can be gained from the Equation (4), which indicates that the Crossover rate is self-adaptive with the fitness of population.

$$P_c = \begin{cases} \frac{k_c(f_{max} - f)}{f_{max} - \bar{f}} & f > \bar{f} \\ \frac{k_c}{f_{max} - \bar{f}} & f < \bar{f} \end{cases} \quad \text{(2)}$$

Where $f_{max}$ is the maximal fitness in currently generation; $\bar{f}$ denote average fitness; $f$ is the fitness of the parent individual of crossover operation; $k_c$ is constant, here $k_c=1.0$.

According to the Equation (2), the solution with less fitness has greater chance to executing local search for finding a better child solution. Whereas the individual possess better fitness get the chance with less probability. Therefore the difference of fitness among population is decreased, and the situation getting into relative extremum may be avoided. The advantages of the policy are keeping the population diversity and preventing premature convergence.

After the crossover progress, if the better solution is not found, the parent should be changed by the mutation operation with mutation rate. In LSHGA, the mutation rate is a function of the population diversity. A conception is introduced as a measure of the population diversity.

The Mutation rate $P_m$ is confirmed by the Equation (3):

$$P_m = \begin{cases} \frac{km(D^t_{min}(S,T))}{2N} & f = f' \\ 0 & f > f' \end{cases}$$

Where, $S$ denote the child individual which come from mutation operation; $T$ is the tour whose distance to $S$ is minimum compared with all other nodes, $S \neq T$, and $T \in J(t)$, $J(t)$ is the set all chromosome in the $t$th generation; denotes the minimal distance from $T$ to $S$; $km$ is constant, here $km=0.5$; $f$ is the fitness of $S$, $f'$ is the fitness of the parent of $S$. After mutation, if the fitness of child chromosome $S$ is not improved, and the difference of $S$ and the other individual is relative big value, the solution $S$ may have run into local extreme. Therefore, according as Equation (3), the $P_m$ becomes a greater number for making $S$ to escape from local optima. While the algorithm is near of the end phase, the difference among population trail off, i.e., the value of $D^t_{min}(S,T)$ reduces. So the $P_m$ decrease too, and making the algorithm convergent quickly.

The reservation ratio $P_e$ is given in Equation (4):

$$P_e = ke \frac{f^2 - f'^2}{f'^2_{max} - f^2}$$

Where $ke$ is constant, here $ke=0.1$; the signification of other parameter is same as in Equation (2). According to the Equation (4), while the variance of the fitness of whole population is greater value, $P_e$ is also relative great value, i.e., more individual whose fitness is better is survival into the next generation. The convergence speed of LSHGA is enhanced. On the other hand, while the variance of the fitness of whole population is smaller value, $P_e$ becomes a lesser value, preserving population diversity, evading plunge into local minimum.

2.5. Initial population

Initial population is yielded randomly accord with uniform distribution in LSHGA. So these individuals scatter the solution space, ensuring population diversity, avoiding getting into relative extreme.

III. RESULTS AND COMPARISON

In this section, we present the experimental results on eight symmetric traveling salesman problems in TSPLIB [10] and compare LSHGA to GGA [11] with 3 TSPs, which belong to the eight problems. We implement the LSHGA on 1.7GHz Pentium M CPU running Matlab 7.0. LSHGA is performed 20 independent runs respectively to all the eight problems. For
evaluating the quality of solution with LSHGA, relative error is made use of and denoted by $\Theta$ and it is given by the equation:

$$\Theta = \frac{C-C^*}{C^*} \times 100\%.$$  \hfill (5)

Where $C$ is the solution of LSHGA; $C^*$ is the solution in [10]. The solutions of 8 TSP instance with LSHGA and the parameters of the algorithm are listed in table 1. Moreover the experimental result compared with GGA is showed in table 2. In the table 1 and table 2, notations, QS, BS, AS and PN, denotes the optimal result in the TSPLIB, the best solution of the approach, the population size of the algorithm and the convergence iteration number respectively. While the scale of problem increases, the dimension of feasible solution space extends exponentially. However, as long as the population size of LSHGA is augmented reasonably, the satisfied result will be gained, and the convergent iteration number only increases finitely. For the eight problems in table 1, all relative error is less than 1.7%. The result demonstrates clearly that LSHGA is effective for solving some TSP. In table 2, it is obviously that the solutions and the convergence number of LSHGA are better apparently than the ones of the GGA for problem Ct42, Ct67 and Ct150. In figure 1 to figure 3, the best solution tours of the LSHGA among 20 independent executions are illustrated for TSP instance Ch120, kroA150 and kroA200 respectively.

### Table 1. The result of solution with LSHGA

<table>
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<tr>
<th></th>
<th>QS</th>
<th>PN</th>
<th>AS</th>
<th>BS</th>
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<tr>
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<tr>
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<td>26254</td>
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<tr>
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<td>29682</td>
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### Table 2. Comparison between LSHGA and GGA based on symmetric TSP

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### IV. CONCLUSION AND SUGGESTION

In this study, we design a novel neighborhood structure for TSP, and the Local Search approach combine with GA to form the LSHGA algorithm. The LSHGA algorithm utilizes the properties of many individual co evolutionary and parallel search of GA, and conquers the defect of local search, which is easily to get into relative optimum. Some effectual ideal are proposed for preserving the population diversity, preventing precocity, and enhancing the speed of convergence. By and large, the algorithm can achieve satisfied solutions, and accomplish convergence quickly for middling-scale TSP. However, the run speed is comparatively slow in the case of large-size TSP.

**References**

AUTHOR(S) PROFILE

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