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Designing a State Feedback Controller for the Two Tank System

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Abstract: This paper describes the method of designing a state feedback controller to place the poles at the required locations. The state feedback controller is the row vector that should be connected in the feedback line of the plant system. This was done for the two-tank system to place its poles at (-3 and -4) location in the s-plane. The other characteristics especially the steady state error was improved using an integrator. The results obtained were good enough satisfy the desired design.

Keywords: state feedback; pole placement; transition matrix; feedback gain; eigenvalues

I. INTRODUCTION

There are many tools and method used to achieve the desired performance of control system. Using the phase variable canonical form of the system is one of these techniques used in control system. It allows the designer to design and test the state feedback controller (estimator). An nth order feedback system has an nth- order closed loop characteristic equation of the form:

$$s^n + a_{n-1}s^{n-1} + \cdots + a_1 s + a_0 = 0 \qquad (1)$$

There are n coefficients whose values determine the system's closed loop poles locations [1].

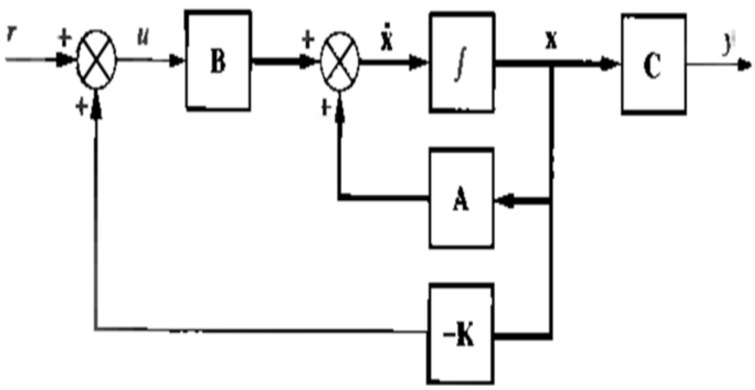

Fig. 1 Plant (in state space form) with state-feedback

If each state variable is fed back to the control u through gain k_i there would be nth gains k_i that could be adjusted to yield the required closed loop pole values. The feedback through k_i is represented as shown in the previous figure. The design of state -variable feedback for closed loop **pole placement** consists of equating the characteristic equation of a closed loop system to a desired characteristic equation and then finding the values of the feedback gains K_i [2].

Open & closed loop state-space Pole Placement using feedback state controller:

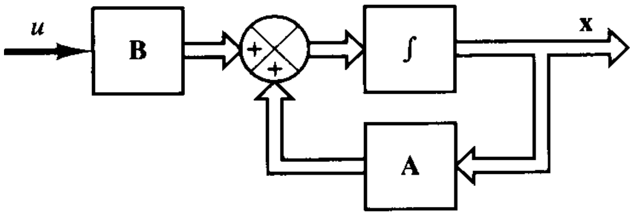

Fig. 2 Open-loop control system

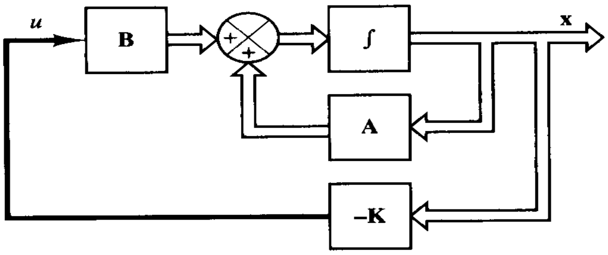

Fig. 3 Closed-loop control system with u=-Kx

The 1xn k matrix is called the state-feedback gain matrix.

II. State Feedback Method

The general form of the state space system consists of the input and output equation in a matrix form.

For the open-loop system

$$X^{\cdot} = Ax + Bu \qquad (2)$$

In feedback system

$$u = -Kx \qquad (3)$$

Then the equation becomes

$$X^{\cdot}(t) = (A - BK)x(t) \qquad (4)$$

The stability and transient-response characteristics are then determined by the eigenvalues of matrix (A-BK) instead of (A). If k is chosen properly, the A-BK can be made a stable matrix [3]. (Hence the system)

For closed-loop the system dynamics is:

$$x^{\cdot}(t) = Ax(t) + Bu(t) \qquad (5)$$

$$y(t) = Cx(t) + Du(t) \qquad (6)$$

The solution of the equation is

$$x(t) = e^{A\,t}x(0) \qquad (7)$$

$$x(t) = e^{(A-BK)t}x(0) \qquad (8)$$

Characteristic equation of the closed-loop system is:

$$\det\,(A - I\lambda) = 0 \qquad \det(A - BK - I\lambda) = 0 \qquad (9)$$

$(sI - A)$ Ch=0q=det

$$|(sI - A)| = 0 \qquad (10)$$

For a given system we want to find the matrix K such that the eigenvalues of the closed loop system are placed in pre-specified positions $\lambda_1, \dots \dots \dots, \lambda_n$.

System poles given by eigenvalues of A and we want to use input u (t) to change the dynamics. We will assume the form of LINEAR STATE FEEDBACK [4].

$$u(t) = r(t) - Kx(t), \quad K \in R^{n\times 1} \tag{11}$$

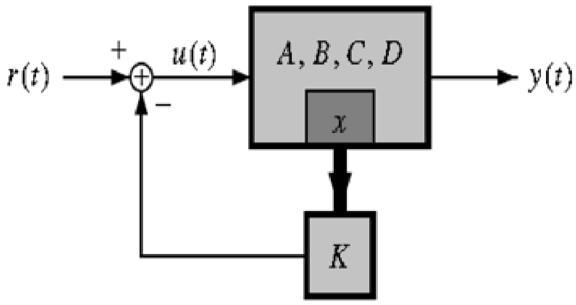

Fig. 4 Full state feedback with gain vector K for two tank system

A typical form of a 2-tank system transfer function:

$$TF(tt) = \frac{0.04}{36s^2 + 12s + 0.51}$$

The 2-tank TF could be represented in state space form as:

$$A = \begin{bmatrix} -0.333 & -0.113 \\ 0.125 & 0 \end{bmatrix} \qquad B = \begin{bmatrix} 0.125 \\ 0 \end{bmatrix} \qquad C = [0 \quad 0.071\,] \qquad D = 0$$

$$x^{\cdot}(t) = \begin{bmatrix} -0.333 & -0.113 \\ 0.125 & 0 \end{bmatrix} x(t) + \begin{bmatrix} 0.125 \\ 0 \end{bmatrix} u(t)$$

$$y(t) = [0 \quad 0.071\,]x(t)$$

For open-loop system

$$\det(sI - A) = (s + 0.333\,)(s) + 0.014 \; = s^2 + 0.333\,s + 0.014$$

For closed-loop

$$u(t) = -[k_1 \quad k_2]x(t) = -Kx(t) \tag{12}$$

$$A_{cl} = A - BK = \begin{bmatrix} -0.333 & -0.113 \\ 0.125 & 0 \end{bmatrix} - \begin{bmatrix} 0.125 \\ 0 \end{bmatrix} [k_1 \quad k_2]$$

$$= \begin{bmatrix} -0.333 - 0.125k_1 & -0.113 - 0.125k_2 \\ 0.125 & 0 \end{bmatrix}$$

So, $\det(sI - A_{cl}) = s^2 + (0.333\; + 0.125k_1)s - (0.014\; + 0.015k_2)$

With $\det(sI - A_{cl}) = s^2 + (0.333\; + 0.125k_1)s - (0.014\; + 0.015k_2)$

So, $0.333\; + 0.125k_1 = 7, \quad or, \qquad k_1 = 53.34$

$0.014\; + 0.015k_2 = 12, \quad or, \quad k_2 = 799.07$

$K = [53.34 \quad 799.07]$

By choosing k_1 and k_2 , we can put eig (A_{cl}) ANYWHERE in the complex plane.

III. RESULTS

The pole placement method usually used when the designer wants to ensure the stability of the system by choosing poles values that lay on the left hand side of the s-plane far away from the marginal vertical line (imaginary axis).

Solving the problem of the poles locations by pole placement is not the end of the story. The characteristics of the system response such as study state error, rise time, and settling time may get worse. This needs to insert anther controller to overcome these problems. Referring to figure (5) which represents the step response of the closed-loop system after adding the required two poles (-3 and -4), one may note that the study state error is big. As known adding an integrator will solve this problem. This is possible and will give a good result. Adding the same integrator without pole placement may cause bad closed-loop response as shown in figure (6).

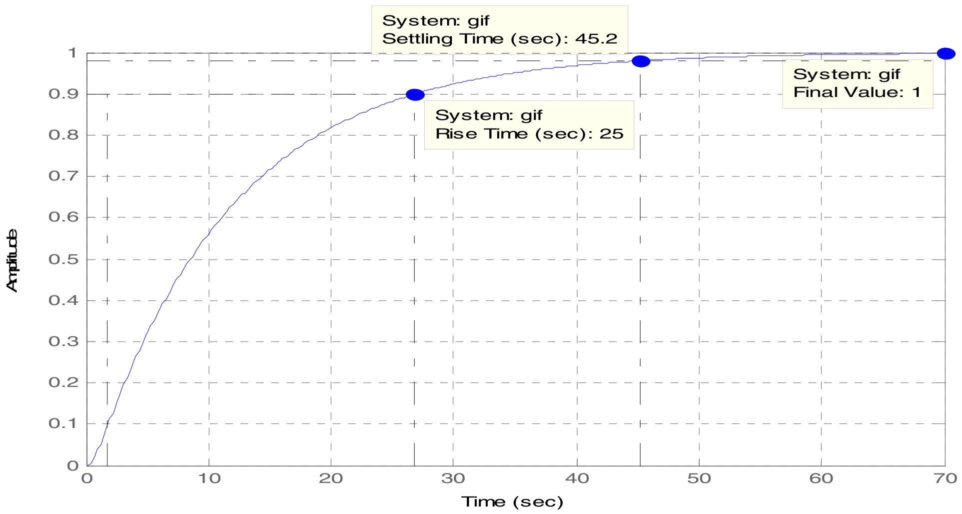

Fig. 5 The step response of the closed-loop system for two-tank system after adding the required two poles

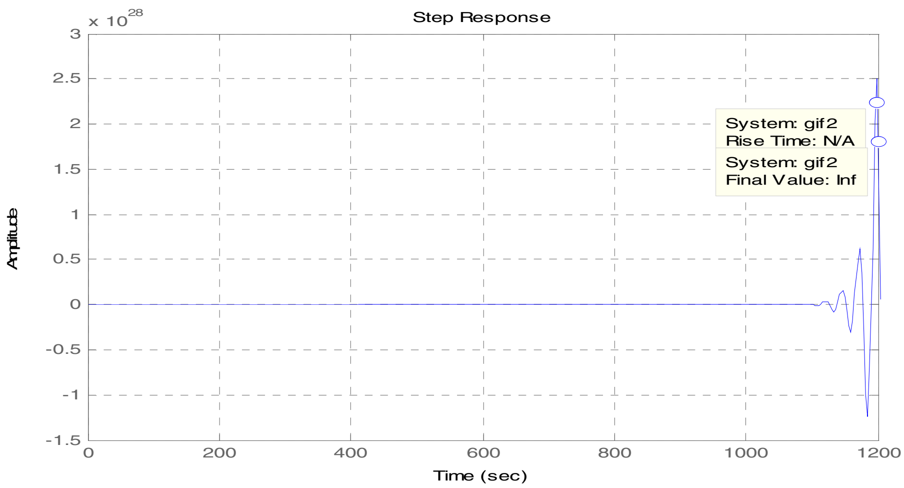

Fig. 6 The step response of the two-tank system before adding the required two poles

Referring to figure (7) the required poles (-3 and -4) are located in their positions beside the poor original poles (-0.3 and -0.05) to show that the original system before adding the state feedback controller is considered as a marginal stability system because its poles almost lay on the vertical line between stability an instability region in the s- plane.

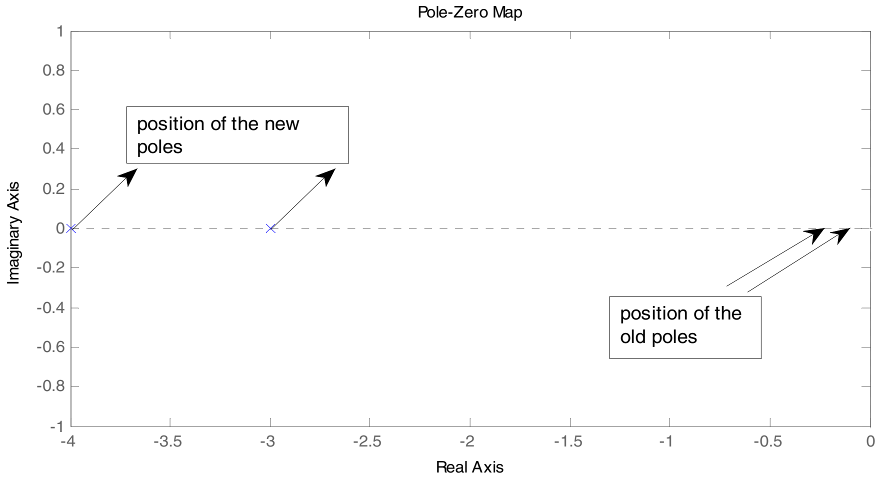

Fig. 7 Position of the poles for two-tank system

IV. CONCLUSION

The summary of this paper was to find a mathematical model of the two-tank system and calculate the transfer function of the system controller. The transfer function was tested using the method of pole placement and calculated the value of gain in feedback system in order to guarantee the most important performance characteristics of this system such as rise time, settling time, and steady state error. The two poles were placed at locations that ensure a good relative stability and good response characteristics.

References

1. Hugn Jack, "Dynamic Systems Model", Draft version 2, 4, 2003.
2. Roland S.Burns, "Advanced Control Engineering", Butter worth Heinemann, 2001.
3. D. Roy Choudhury, "Modern Control Engineering", Prentice-Hall of India, New Delhi, 2005.
4. Katsuhiko Ogata," Modern Control Engineering ", Prentice-Hall, Inc., 1997.
5. S.JANANI, C.YASOTHA, "design and analysis of state feedback controller using pole placement technique", International Journal of Engineering Science and Innovative Technology (IJESIT) Volume 3, Issue 1, January 2014.