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## *A Novel Sallen-Key Compensator used with a highly Oscillating Second-Order Process*

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*Abstract: Compensators are used in place of classical PID controllers for possible achievement of better performance. Highly oscillating processes require more effort in selecting proper controllers or compensators.*

*In this paper a novel compensator based on the a type of Sallen-Key filters is proposed and applied to control a process having 85 % overshoot. The compensator proposed is capable of controlling the steady-state characteristics of the closed-loop control system and its dynamic characteristics. The advantage of the proposed compensator is its simple tuning without need to advanced optimization technique. It was possible with the Sallen-Key compensator to satisfy a system performance without any overshoot with a settling time of about 9.3 seconds and steady-state error as low as 0.01 or less for a unit step input.*

*Keywords: Highly oscillating processes ; compensators ; Sallen-Key filter ; Sallen-Key compensator ;control system performance.*

### I. INTRODUCTION

Feedforward compensators find wide application in both linear and nonlinear dynamic systems. The design of classical compensators such as lag, lead, lag-lead, PID and pre-filter are investigated in automatic control textbooks [1-5].

Giusto and Paganini (1999) considered the design of feedforward compensator for robust  $H_\infty$  or  $H_2$  performance under structured uncertainty [6]. Ro, Shim and Jeong (2000) developed a PD control scheme with a nonlinear friction estimate algorithm for a ball-screw-driven slide system. Their approach resulted in a system performance with steady-state error under 1.5 % [7]. Messner and McNob (2001) presented a method for the design of compensators for linear time-invariant dual-input / single-output system. They reduced the problem to 2-single input / single output design problems [8]. Shi, Dimirovski, Zhang and Stankovski (2002) introduced a theoretical approach generating a new design method. They designed normal dynamic compensators such that the closed-loop control system is asymptotically stable and has no impulse effect [9]. Choi, Son, Han and Choi (2003) presented a micro-positioning mechanism with dual servo system. They used a lead compensator for the control of the system [10]. Gessing (2004) used Smith compensator connected in parallel to the plant. The proposed approach is expected to simplify the control system design and improve the control accuracy [11]. Wu and Duan (2005) addressed the design of a type of dynamical compensators for matrix second order linear systems in the matrix framework using a complete parametric approach [12].

Leva and Bascetta (2006) presented a methodology to design the feedforward path of 2DOF regulators for optimum set point tracking using a nonparametric model of the control loop identified online [13]. Panda and Padhy (2007) applied the genetic algorithm optimization technique to design a thyristor controlled series compensator to enhance the power system stability using a lead-lag and a PID compensators [14]. Rico and Camacho (2008) presented a review of the main dead-time compensators designed to improve the closed loop characteristics and to control unstable systems [15]. Shu'aibn and Adamu

(2009) presented the design of a phase-lead compensator (PD-controller) of a magnetic levitation system using the root-locus method and the MATLAB control toolbox [16]. Roy, Wan, Saberi and Malek (2010) presented a methodology for designing low gain linear time-invariant compensators for semi-global stabilization using a pre-compensator plus a static output feedback [17]. Zanasi, Cuoghi and Ntogramatzidis (2011) presented the dynamic structure and the control properties of a new form of lead-lag compensator with complex zeros and poles [18]. Mori (2012) investigated the 2-stage compensator design of linear systems. He derives various types of the 2-stage compensator design with partial feedbacks [19]. Espi (2013) presented an optimal capacitance design for the DC link of a back-to-back converter for wind turbine power generation. He used PI-compensator for the voltage regulation [20]. Hassaan (2014) proposed specific types of controllers and compensators to suppress the high oscillations of the second-order-like processes [21,22].

## II. ANALYSIS

### Process

The process is a second order process having the transfer function,  $G_p(s)$ :

$$G_p(s) = \omega_{np}^2 / (s^2 + 2\zeta_p \omega_{np} s + \omega_{np}^2) \quad (1)$$

Where:

$$\omega_{np} = \text{process natural frequency} = 10 \quad \text{rad/s.}$$

$$\zeta_p = \text{process damping ratio} = 0.05$$

The time response of this process in a unit feedback loop without compensation to a unit step input is shown in Fig.1 as generated by MATLAB:

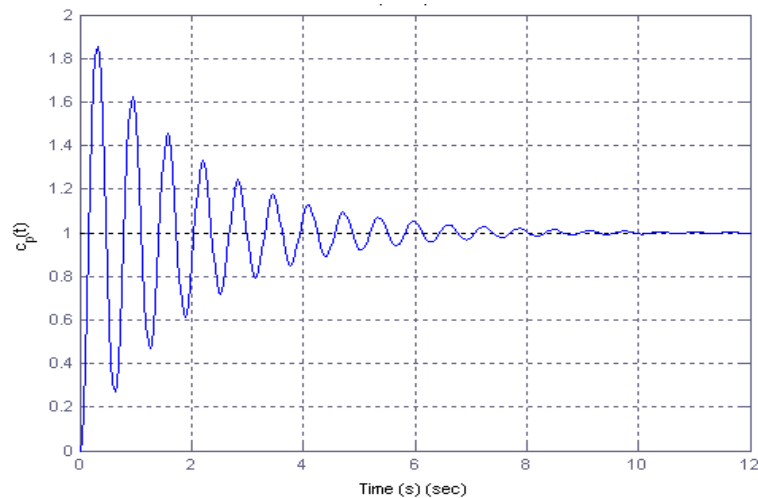


Fig.1 Step response of the uncompensated process.

The performance of the process is measured by its maximum percentage overshoot and its settling time. It has a maximum overshoot of 85.45 % and about 6 seconds settling time.

### The Sallen-Key filter:

Sallen and Key presented a practical method for the design of RC active filters having different transfer functions for low-pass, band-pass and high-pass filters [23].

A second order Sallen-Key low pass filter has a transfer function,  $G_f(s)$  given as [24]:

$$G_f(s) = K \omega_{nf}^2 / (s^2 + 2\zeta_f \omega_{nf} s + \omega_{nf}^2) \quad (2)$$

It has the 3 parameters:

- Gain, K.
- Natural frequency,  $\omega_{nf}$
- Damping ratio,  $\zeta_f$

The three parameters are function of the values of the resistance and capacitance of the resistor and capacitor components encountered in the filter electronic circuit [24].

#### A Sallen-Key compensator:

A Sallen-Key filter compensator consists of the classical low-pass second-order filter of Eq.2 The gain K of the filter is suitable to control the steady-state characteristics of the closed-loop control system incorporating the feedforward compensator and the process. The compensator has the transfer function,  $G_c(s)$ :

$$G_c(s) = K \omega_{nf}^2 / (s^2 + 2\zeta_f \omega_{nf} s + \omega_{nf}^2) \quad (3)$$

#### Control system transfer function:

Assuming that the control system is a unit feedback one, its transfer function with  $G_c(s)$  of Eq.3 and  $G_p(s)$  of Eq.1 is:

$$M(s) = b_0 / (s^4 + a_1 s^3 + a_2 s^2 + a_3 s + a_4) \quad (4)$$

where:

$$b_0 = K \omega_{nf}^2 \omega_{np}^2$$

$$a_1 = 2\zeta_p \omega_{np}$$

$$a_2 = \omega_{np}^2 + 4\zeta_f \zeta_p \omega_{nf} \omega_{np} + \omega_{nf}^2$$

$$a_3 = 2\zeta_f \omega_{nf} \omega_{np}^2 + 2\zeta_p \omega_{np} \omega_{nf}^2$$

$$a_4 = \omega_{nf}^2 \omega_{np}^2 (1 + K)$$

#### Stability of the closed-loop control system:

The compensator parameters have to be determined such that the closed loop control system is stable. Since the closed-loop system is a fourth order one, it is possible to be stable. Therefore, the compensator parameters have to match the stability conditions of the control system. Using the characteristic equation which is the denominator of Eq.4 and the Routh-Hurwitz stability criterion [25], the stability conditions are:

$$\text{Condition 1: } f_1 = (a_1 a_2 - a_3) / a_1 > 0 \quad (5)$$

$$\text{Condition 2: } f_2 = (f_1 a_3 - a_1 a_4) / f_1 > 0 \quad (6)$$

Eqs.5 and 6 play an important rule in tuning the Sallen-Key compensator. For example let: K = 99 (for an 0.01 steady-state error) the maximum value of the compensator natural frequency (in rad/s) against the compensator damping ratio is shown in Fig.2.

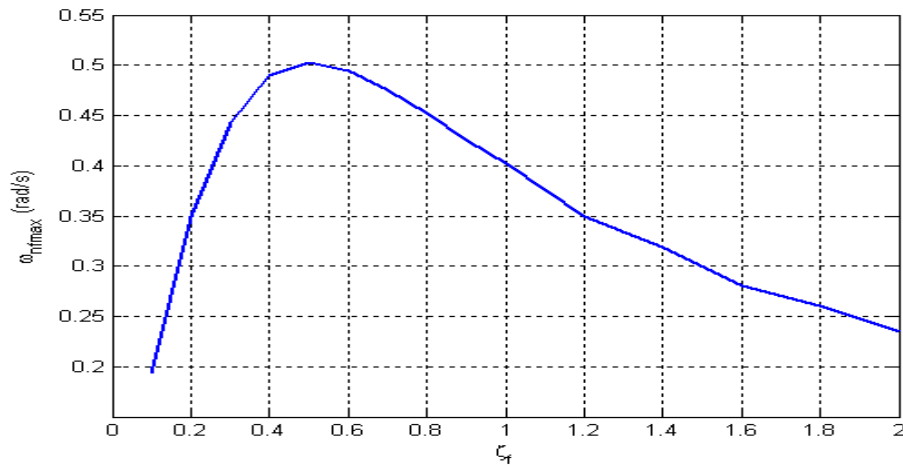


Fig.2 Maximum compensator frequency for a stable control system

### System step response and performance:

A unit step response is generated by MATLAB using the numerator and denominator of Eq. 4 providing the system time response  $c(t)$  as function of time for a set of compensator parameters [26].

The characteristics of the compensated control system quantifying its performance are:

- Steady-state response,  $c_{ss}$ :

Using Eq.4, the system steady-state response for a unit step input is:

$$c_{ss} = K / (1+K) \quad (7)$$

- Steady-state error,  $e_{ss}$ :

Using Eq.4, the system steady-state error for a unit step input is:

$$e_{ss} = 1 / (1+K) \quad (8)$$

- Maximum percentage overshoot,  $OS_{max}$ :

Using the time response of the control system to a unit-step input, the maximum percentage overshoot is:

$$OS_{max} = 100 (c_{mas} - c_{ss}) / c_{ss} \quad (9)$$

Where:  $c_{max}$  = maximum time response to a step input.

$c_{ss}$  = steady-state response of the control system to the unit step input

- Settling time,  $T_s$ :

The time response of the system enters a band of  $\pm 5\%$  of the steady-state response and remains inside this band.

### III. SALLEN-KEY COMPENSATOR TUNING

The compensator proposed in this work is tuned manually without need to sophisticated optimization techniques. The procedure is as follows:

- Assign the compensator gain  $K$  according to the desired steady-state error using Eq.7.
- Assign a value for  $\zeta_f$  (filter damping ratio).
- Using MATLAB, plot  $f_1$  and  $f_2$  against  $\omega_{nf}$  using Eqs.5 and 6 . This step determines the maximum value of  $\omega_{nf}$  ( $\omega_{nfmax}$ ) for a stable control system.

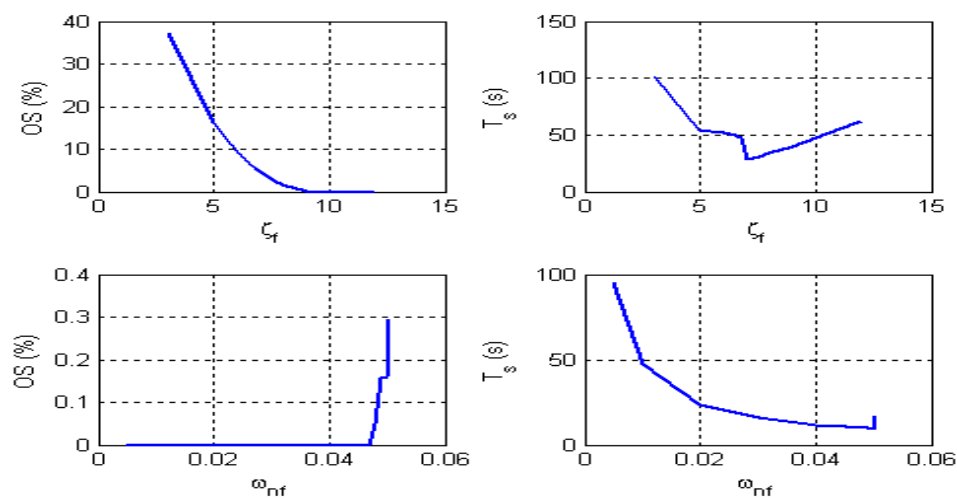
- Discretize the range 0 to  $\omega_{nfmax}$ .
- For each set of values for  $K$ ,  $\zeta_f$  and  $\omega_{nf}$  extract the overshoot and settling time using the command "stepinfo".
- Repeat for different values of  $\zeta_f$ .
- Select the appropriate combination of the compensator parameters satisfying the desired performance of the closed loop control system.
- The application of this procedure produces the  $\omega_{nfmax}$  using the highly oscillating process of Fig.1 as in Table 1:

**TABLE 1**  
 **$\omega_{nfmax}$  against  $\zeta_f$  for the Sallen-Key compensator ( $K = 99$ )**

$\zeta_f$	0.2	0.4	0.6	0.8	1
$\omega_{nfmax}$	0.35	0.49	0.495	0.453	0.402
$\zeta_f$	1.5	2	4	6	8
$\omega_{nfmax}$	0.335	0.235	0.126	0.082	0.063

#### IV. SALLEN –KEY COMPENSATOR APPLICATION

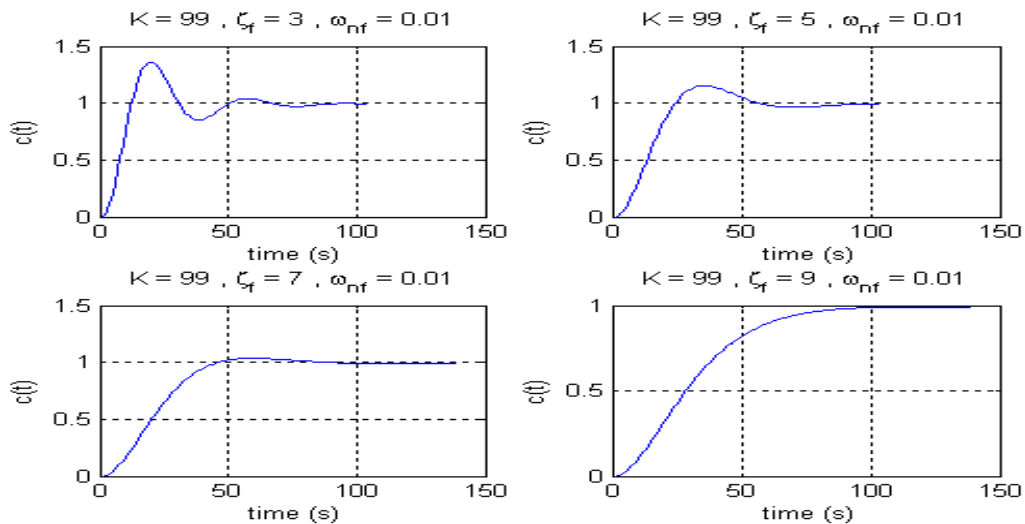
The effectiveness of this new compensator is examined through the highly oscillating second order process whose step response is shown in Fig.1. The effect of compensator damping ratio and natural frequency on the control system performance in terms of its maximum percentage overshoot and settling time for  $K = 99$  is shown in Fig.3.



*Fig.3 Effect of filter damping ratio and natural frequency on control system performance.*

It is clear from Fig.3 that the overshoot drops to zero (compared with 85 % of the process). The settling time can drop to 9.3 seconds (compared with 6 seconds of the process).

The step time response of the closed-loop system incorporating the proposed compensator and the process is shown in Fig.4 for a set of the Sallen-Key compensator parameters.



**Fig.4 Step time response of the control system for a set of compensator parameters.**

- K is kept at 99 for an 0.01 steady-state error.
- $\omega_{nf}$  is kept at 0.01 rad/s.
- The control system performance improves as the compensator damping ratio increases.
- It is possible to get a time response to a step input without any oscillations.

#### V. REVIEW OF RESEARCH AND DEVELOPMENT IN THE SUBJECT

- Highly oscillating processes represent a big industrial problem due to its side effects on the final product of some industries.
- A new Sallen-Key filter-based forward compensator is proposed.
- The adjustable gain of the compensator is used to control the steady-state characteristics of the control system.
- A manual tuning approach is used.
- It was possible using the proposed compensator to produce a zero overshoot control system when using a process of 85 % maximum overshoot.
- The highly oscillating process had about 6 seconds settling time. Using the proposed compensator with the manual tuning suggested, it was not possible to reduce the settling time below 9.33 seconds.
- It may be possible to reduce the settling time of the control system if sophisticated tuning techniques are used.

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