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## *Hurst Exponent of BSE Sensex*

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*Abstract: Efficient market hypothesis states that markets are inherently efficient and Stock market returns are Independent and identical distributed (IID). If the returns are IID then it will form a bell curve. To analyze this daily returns of 4096 days return are used. The curve obtained is leptokurtic. It shows that the returns are not normally distributed. Hurst exponent is a statistical tool that is used to find dependency in time series. The result contradicts the predictions made by efficient market hypothesis. The result obtained is very similar to Fibonacci ratio  $\Phi$ ; whether that is a mere coincidence or it has some deeper meaning is left for future research.*

*Keywords: Stock market; Efficient Market hypothesis; Time Series; Hurst Exponent, Power Law.*

### I. INTRODUCTION

Efficient Market Hypothesis proposes prices in the financial markets are inherently fair and the price of an asset reflects all the information. The term was coined by E F Fama in a paper 'On the average; competition will cause the full effects of new information on intrinsic values to be reflected "instantaneously" in actual prices'. The efficient market hypothesis is associated with the idea of a "random walk," which says that the price fluctuations are entirely random that is you cannot predict the future price from analyzing the past information. The random character of stock market is first modeled by Louis Bachelier in his 1900 PhD thesis, "The Theory of Speculation". He was a PhD Scholar under the guidance of Henri Poincaré the great French mathematician, His work was largely ignored until the 1950s.

The logic of the random walk idea is that if the flow of information is unimpeded and information is immediately reflected in stock prices, then tomorrow's price variation will reflect only tomorrow's news and will be independent of the price changes today. But news and the information they carry are unpredictable and, thus, resulting price fluctuation must be unpredictable and random. So market prices reflects all the known information and even a naïve investor who don't know much about investing picks a portfolio randomly from stock market makes a profit similar to an expert fund manager. There are three forms of Efficient Market Hypothesis – (i) Weak, (ii) Semi Strong and (iii) Strong

**Weak Form of efficiency**, asserts that the current price fully incorporates information contained in the past history of prices only.

**Semi Strong Form of efficiency**, states that all publicly known and available information is quickly incorporated into the stock price. This implies that an investor cannot act on new public information and expect to earn above-average returns.

**Strong Form of efficiency**, states that stock prices fully reflect all information whether it is public or nonpublic (inside information), which means that insiders cannot make abnormal profits in the market. The logic for strong form is that market predicts future developments about the price and therefore the stock price may have incorporated the information and evaluated in a much more objective and informative way than the insiders.

The underlying mathematical theory in which the Efficient Market Hypothesis is established is through **Central Limit Theorem**, states that as a sample of mutually independent random numbers approaches infinity, the probability density function approaches the normal distribution curve or the Gaussian bell curve. First, this implies that the Efficient Market Hypothesis believes that market changes are random; and if the market changes are plotted over a period of time, they should construct the normal curve.

## II. RESEARCH METHODOLOGY

The researcher has used close price of BSE Sensex from January 1991 to February 2008 it has 4097 days closing value which is taken for analysis. If the stock prices are random then it will produce a normal bell curve. Hurst exponent is a statistical tool that used to find out any underlying trends in the time series.

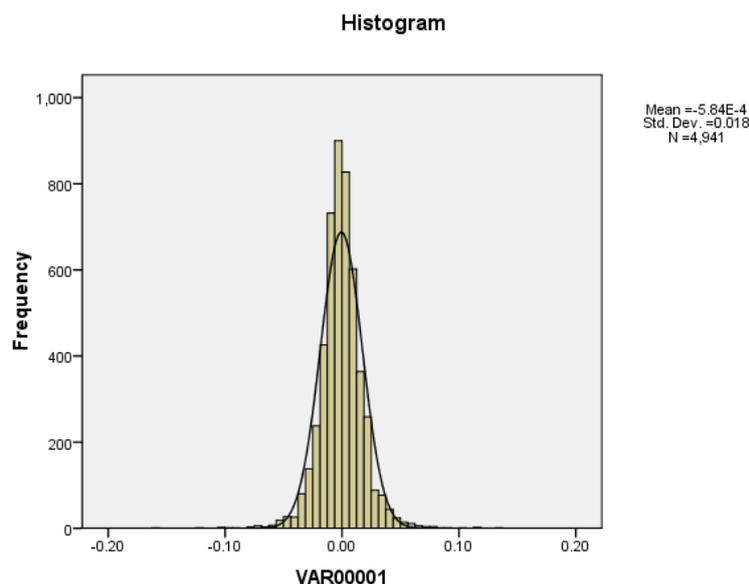
### Skewness and kurtosis

BSE sensex daily log returns from January 1991 to August 2011 are used for analysis. Skewness and kurtosis is the measure of change in the normal distribution the skewness is used to check that the evenly distributed to left and right of the mean and the skewness value thus obtained is -0.035123638. It shows that the distribution is slightly skewed. The kurtosis value is 5.590558125 the value is above 3 shows it is **leptokurtic**. It shows that more of the value is crowded around the mean of the distribution and few large deviations from the mean are affecting the standard deviation.

### Bell curve

Fig 3 is the histogram of daily log returns of BSE sensex from January 1991 to August 2011 and the graph is not looking like the bell curve. So we have to discard the idea that price changes are random.

Fig 1. . Histogram of BSE sensex daily log returns from January 1991 to August 2011 It is drawn using SPSS



## III. HURST EXPONENT

Hurst exponent is a statistical tool that is used to calculate the fractal dimension of time series. Fractals are geometric shapes that have a self similarity. The whole look like the parts. Financial time series has memory. They are not random as proclaimed by efficient market hypothesis and Martingale process. Hurst exponent is developed in early 20<sup>th</sup> century by for constructing dam across Nile River. If the dam is too small then it is can't stop the flooding and too big it is a waste of resources. He stared with 800 year of Nile flooding history estimate Hurst exponent.

To estimate the Hurst exponent, one must first estimate the dependence of the rescaled range on the time span  $n$  of observation. A time series of full length  $N$  is divided into a number of shorter time series of length  $n = N, N/2, N/4 \dots$ . The average rescaled range is then calculated for each value of  $n$ .

For a (partial) time series of length  $n$ ,  $X = X_1, X_2, \dots, X_n$ , the rescaled range is calculated as follows:

1. Calculate the mean;

$$m = \frac{1}{n} \sum_{i=1}^n X_i.$$

2. Create a mean-adjusted series;

$$Y_t = X_t - m \quad \text{for } t = 1, 2, \dots, n.$$

3. Calculate the cumulative deviate series  $Z$ ;

$$Z_t = \sum_{i=1}^t Y_i \quad \text{for } t = 1, 2, \dots, n.$$

4. Compute the range  $R$ ;

$$R(n) = \max(Z_1, Z_2, \dots, Z_n) - \min(Z_1, Z_2, \dots, Z_n).$$

5. Compute the standard deviation  $S$ ;

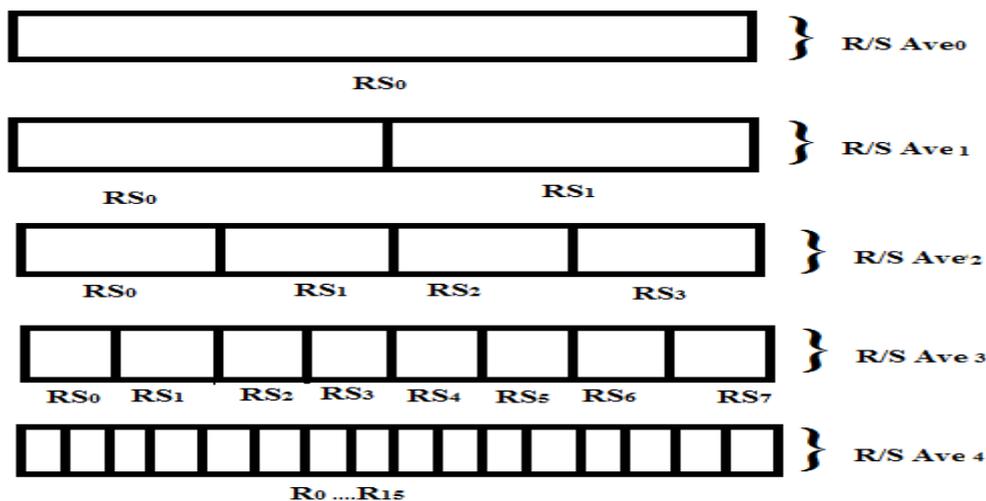
$$S(n) = \sqrt{\frac{1}{n} \sum_{i=1}^n (X_i - m)^2}.$$

6. Calculate the R/S (rescaled range)  $R(n)/S(n)$  and average over all the partial time series of length  $n$ . The Hurst exponent is estimated by fitting the power law to the data.

$$E \left[ \frac{R(n)}{S(n)} \right] = Cn^H$$

This can be done by fitting a straight line, with X axis as  $\log_2(\text{Rescale Range average})$  and Y axis as  $\log_2(n)$  ( $n$ =length of partial time series); the slope of the line gives  $H$ . Such a graph is called a *pox plot*.

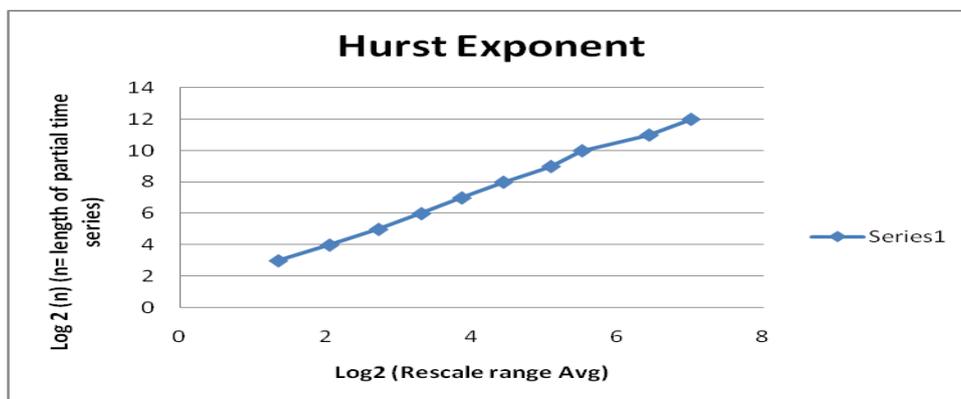
As shown in Figure 1, the rescaled range is calculated for the entire data set (here  $\text{Save}_0 = RS_0$ ). Then the rescaled range is calculated for the two halves of the data set, resulting in  $RS_0$  and  $RS_1$ . These two values are averaged, resulting in  $\text{RSave}_1$ . In this case the process continues by dividing each of the previous sections in half and calculating the rescaled range for each new section. Calculate average for each section. It can be continued till eight data points. Beyond that data set is too small for any reliable calculation.



n	Rescale Range Average	log <sub>2</sub> (Rescale Range Average)	Log <sub>2</sub> (n)(n=length of partial time series)
8	2.5365792	1.342884207	3
16	4.1308526	2.046439582	4
32	6.5876531	2.719764586	5
64	9.8981751	3.307162564	6
128	14.537075	3.861665109	7
256	21.609307	4.433580901	8
512	33.958794	5.085713319	9
1024	45.705713	5.514302602	10
2048	86.464125	6.43402976	11
4096	128.80668	7.009063604	12

Source: 4096 daily return of BSE sensx from January 1991 to February 2008 has been used to calculate rescale range. Matlab software is used to calculate rescale range algorithm

Graph (1) Regression line for estimating Hurst exponent



The table(1) shows the Rescale Range for region size 8 to 4096. Hurst Exponent can be obtained by plotting graph of log<sub>2</sub> (RS average) on X axis and log<sub>2</sub> (n) on Y axis. **Hurst exponent is the slope of the regression line in graph (1) H= 0.615691**

**Interpretation of the result**

Hurst exponent value  $H < 0.5$  means it is an anti persistent series.

Hurst exponent value  $H=0.5$  means it is a random series it follows brown motion.

Hurst exponent value  $H>0.5$  means it is a persistent series

Since Hurst exponent is above 0.5,  $H = (0.615691)$  it is **showing persistent series**.

The result contradicts the random price movement predicted by Efficient Market Hypothesis and it shows that there is trend in the price movement. That is validating the technical analysis.

The rescale range is governed by power law equal to  $cn^H$  substituting the value of hurst exponent we get  $cn^{0.615691}$  Which is not equal to  $cn^{0.5}$  as predicted by efficient market hypothesis.

Such a distribution is described as Pareto Levi stable distribution. Since this type of distribution is observed in income distribution of population and the 80:20 rule that is 80% of the sales from 20% of clients. This means there could be an underlying rule that can describe the whole of stock market, Income distribution of people in a country, consumer behavior. Stable distribution has location parameter (mean) and scale parameter (standard deviation). Normal distribution is a type of stable distribution with mean zero and finite standard deviation. This sort of distribution means that there would be too many small deviations from the average as well too many very large deviations. But there is too few of is moderate deviations. The extreme deviations from the mean were of particular interest because those were the cases of stock market booms and busts. Compared with a normal distribution the tail events have higher probability so the tails are much thicker so they are called fat-tailed distribution.

**IV. TECHNICAL ANALYSIS**

The value of hurst exponent is very similar to inverse of Fibonacci ratio  $\Phi$  (phi) but it should be careful while equating  $H=1/\Phi$ . Even though most of the technical analyst traders use Fibonacci retracement to trade research shows that it is not true. It is vague approximation.

**V. UNIFICATION OF SOCIAL SCIENCE**

Newton's Theory of gravity unified earth and heavens that means apple falling to earth and moon going around the earth are of the same force. James Clerk Maxwell unified magnetism and electricity in to electromagnetism. Albert Einstein unified energy and mass ( $E=mc^2$ ). Standard model of physics unified electromagnetism, weak and strong forces. Grand unification theory hopes to unify all the forces of nature electromagnetic, weak force and strong force and Gravity.

Social science areas like economics, management, psychology and sociology etc which are studied as different disciplines with different set of theories must be brought under one single unified theory. This is also referred as axiomising social science. That means we can make predictions using theory and then verify using empirical evidence.

**VI. CONCLUSION**

This is strong empirical evidence in favor of the result. Technical analysis uses past information to find out the trends in the market. This result validates technical analysis by stating that there is dependency in data. The Hurst exponent of 0.61 and the Fibonacci ratio are very similar that it is easily equate with each other. But more research should be done to establish their true relation.

**VII. SCOPE FOR FURTHER RESEARCH**

Since the results shows that there is dependency in the data, chaos theory which studies the non linear dynamics can be applied. Chaotic system shows a periodic behavior even though the underlying rules are simple.

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