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Just in Time Inventory Model with Unknown Demand Rate

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***Abstract:** This paper deals with the purchasing aspect of Just-In-Time (JIT) considering varying setup costs. The various features of the JIT purchasing like frequent deliveries of small shipments, reduction in the inventories are taken into consideration. Here we deal with determination economic order quantity under JIT purchasing having varying setup cost when demand is fixed and varying. In both cases it is observed that with increase in optimum number of shipments results in decrease in total cost. Also increase in contract quantity results in significant decrease in total cost. Also we have developed a model which shows the determination of economic order quantity for perishable product. The perishable product cannot be held for a long time and hence JIT is the best suitable which reduces the amount of inventory and its cost significantly.*

***Keywords:** Exponential smoothing, Taylor's expansion, inventory, JIT.*

I. Introduction

Sales data in disaggregated form (not monthly, quarterly or yearly sales figures) are not always available. Therefore, we use previous order data to estimate the rate of demand in JIT model.

II. Notation

t_i : Interval between the last $(i + 1)^{th}$ and i^{th} order

(Note : it is counted form the past, i.e. t_i is the i^{th} period in the past)

Q_i : Order quantity at the last i^{th} order

(Replenishment order)

l_i : Last i^{th} auxiliary value for D; $l_i = Q_i/t_i$

(Note: The order Q_i is the substitute for the demand before the last ith order)

III. Problem Formulation

In the lot size model it is assumed that D is constant. Hence, it has to be tested whether the observations actually support the assumption. A quick answer can be obtained from a visual inspection of the series $\{l_i\}_{i\in N}$ (where N is set of natural number) If the observations l_i fluctuates around a long-run constant average, then the arithmetic mean from the n existing observations is a suitable estimator for the true D

$$\bar{D} = \frac{1}{n}\sum_{i=1}^{n} l_i \qquad (1)$$

If one chooses only each of the last m observations, m is fixed, and one speaks of a moving average.

If the observations stretch over a longer period, shifts in the demand process, as a rule, will occur because of product mix changes, changes in brand loyalties, etc. It is then meaningful to give more weight to newer data than to old ones. By geometric weighting, one obtains for $n \to \infty$:

$$\bar{D} = \frac{\sum_{i=1}^{\infty} \rho^{i-1} l_i}{\sum_{i=1}^{\infty} \rho^{i-1}} = (1-\rho).\sum_{i=1}^{\infty} \rho^{i-1} l_i ,|\rho| < 1 \qquad (2)$$

ρ is the weight factor.

The advantage of this weight is that it allows $\bar{D}$ being easily computed recursively. It is

$$\bar{D}_{t+1} = (1-\rho) l_1 + \rho \bar{D}_t , t = 1,2,\ldots \qquad (3)$$

We substitute ρ with $1 - \rho$ and obtain the usual terms from time series theory

$$\bar{D} = \rho l_1 + (1-\rho) \bar{D}_1 \qquad (4)$$

l_1 is the last observation, $\bar{D}_1$ the previous and $\bar{D}$ the new estimate for D. The equivalence between (2) and (3) is easily shown by successive solution of the recursion (3).

The estimation procedure (3) or (4) is called a first-order exponential smoothing. The previous values are exponentially damped. Through this, the speed of adaptation to a sudden occurrence of a change in level is increased considerably as compared to the arithmetic mean method. It becomes especially clear if one formulates equation (1) recursively

$$\bar{D}_{t+1} = \frac{1}{t+1} l_1 + \frac{t}{t+1} \bar{D}_t \qquad (5)$$

And let t be very large. The latest observation is taken as the new estimate value with a weight of 1/(t+1). As time increases, this influence always becomes smaller. On the other hand, it remains constant with exponential smoothing.

The theoretical basis of the first order exponential smoothing lies in the modeling of an adaptive expected behavior according to the formula

$$\mathrm{E}\{D_{t+1}\} - \mathrm{E}\{D_t\} = \rho(l_1 - \mathrm{E}\{D_t\}),$$

From which

$$\mathrm{E}\{D_{t+1}\} = \rho l_1 + (1-\rho)\mathrm{E}\{D_t\}$$

It describes the structure of the time series, which fluctuates, about a constant level, where this level is itself disrupted by random displacements.

E {.} is the Expected Value Operator

Exponential smoothing is an appropriate forecasting method with this type of time series. Time series theory, moreover, gives a general statement about the structures of time series for which this forecasting method is optimal. More discussions on this and other refined variations of exponential smoothing can be found in BOX/JENKINS (1976) and MAKRIDAKIS/WHEEL WRIGHT (1987)

The usual value of ρ lie between 0.01 and 0.1. The choice of a suitable value ρ is again in itself a decision problem where the perception about the speed of adaptation comes into play.

IV. PROFIT MAXIMIZATION

We assume that a good is sold at a price p per unit, bought at price 'a' and the other data as before. The objective is profit maximization. The average profit per unit time amounts obviously to

$$g = \frac{pQ - aQ - (A + NP) - \frac{HQ^2}{2ND}}{Q/D}. \qquad (6)$$

If the revenues and costs occurring within an inventory period are divided by the length of the period, then

$$g = D(p-a) - \frac{(A+NP)D}{Q} - \frac{HQ}{2N}$$

$$g = D(p-a) - c \qquad (7)$$

Where c represents, as before, the average cost of inventory per unit time. Furthermore,

$$\underset{Q}{\text{Max}}\; g = D(p-a) + \underset{Q}{\text{Max}}\left(-\frac{(A+NP)D}{Q} - \frac{HQ}{2N}\right)$$

$$= D(p-a) - \underset{Q}{\text{Min}}\left(\frac{(A+NP)D}{Q} + \frac{HQ}{2N}\right) \qquad (8)$$

The profit maximization problem is therefore identical to the cost minimization problem of the standard inventory theory except for the additive constant D(p-a).

V. INVENTORY EVALUATION

A firm has a license to engage in the warehousing business until time period T. Let the current stock be y at a given period t. How large is the commercial value of the firm? In other words, how does one evaluate the inventory y?

The value of the firm is obviously a function of the stock level y as well as the remaining time T-t. It is described by v(y, T – t)

During a short period Δt, it evolves as follows

$$v(y, T-t) = pD\Delta t - \frac{H}{N} y\Delta t + v(y - D\Delta t, T - t - \Delta t), y > 0 \qquad (9)$$

Since the current revenue is pDΔt, the current costs are $\frac{H}{N}y\Delta t$ and stocks are reduced by -DΔt.

If y = 0, then

$$v(0, T-t) = -(A+NP) - aQ + v(Q, T-t), y = 0 \qquad (10)$$

Applies because stocks must be replenished up to Q and that causes the cost (A+NP + aQ).

The Taylor–Approximation for v (y - DΔt, (T – t) - Δt) is

$$v(y - D\Delta t, (T-t) - \Delta t) = v(y, T-t) - v_y \cdot D\Delta t - v_t \cdot \Delta t$$

Substituting in (9) and dividing by Δt result in the partial differential equation for v

$$Dv_y + v_t = Dp - \frac{H}{N} y \qquad (11)$$

With the boundary condition (10) and end value condition

$$v(y, 0) = 0 \qquad (12)$$

So that the end condition will hold, assume that

$$Y(T) = 0$$

That is, a final stock of zero is planned.

It is not unreasonable to attempt to separate the value function into a purely time-dependent and a purely volume-dependent part

$$v(y, T - t) = w(y) + g. (T - t). \qquad (13)$$

In addition, the time-dependent part is set proportional to the remaining time. The proportionality factor is to be interpreted as the rate of profit per unit time. Using the formula (13), the partial differential equation (11) yields

$$D\, w'(y) + g = D\, p - \frac{H}{N} y. \qquad (14)$$

Integrating from 0 to y gives

$$w(y) - w(0) = (p - \frac{g}{D})y - \frac{Hy^2}{2DN} \qquad (15)$$

In particular

$$w(0) = 0.$$

Using the boundary condition (10), one obtains for y = Q,

$$w(Q) - w(0) = (A + NP) + aQ = (p - \frac{g}{D})Q - \frac{HQ^2}{2DN}$$

The rate of profit is then determined to be

$$g = D[p - a - \frac{(A + NP)}{Q} - \frac{HQ}{2DN}] \qquad (16)$$

The profit margin with rate D is

$$p - a - \frac{(A + NP)}{Q} - \frac{HQ}{2DN} = p - a - \bar{c}$$

$\bar{c}$ is the unit cost per time. Substituting (16) and w (0) = 0 in (15) results into the value of inventory y

$$w(y) = \left[a + \frac{A + NP}{Q} + \frac{HQ}{2DN} \right] y - \frac{Hy^2}{2DN} \qquad (17)$$

The value of the business consists of the value of inventory (17) and the value of the remaining time g (T – t). The value of the stock is a quadratic, not a linear nor a proportional, function of the stock. It reaches its maximum at

$$y^* = \frac{DN(a + \frac{(A + NP)}{Q} + \frac{HQ}{2DN})}{H}$$

$$y^* = \frac{DNa}{H} + Q \qquad (18)$$

Using the Wilson lot size formula for Q. The value of the stock increases therefore with the stock in the whole range $0 \le y \le Q$.

If one considers only the added value of stock m (y), that is the surplus above the buying price a, then according to (17) we have

$$m(y) = \left(\frac{A+NP}{Q} + \frac{HQ}{2DN} \right) y - \frac{Hy^2}{2DN} \quad m(y) = \sqrt{\frac{2(A+NP)H}{DN}} \cdot y - \frac{Hy^2}{2DN}. \qquad (19)$$

This added value assumes its maximum if

$$\frac{dm}{dy} = 0 \quad \Rightarrow \sqrt{\frac{2(A+NP)H}{DN}} - \frac{Hy}{DN} = 0$$

$$\text{Implies } y = \sqrt{\frac{2(A+NP)DN}{H}} = Q$$

VI. Conclusion

The optimal order quantity is therefore the one that maximizes the added value of inventory. The evaluation of inventory levels and its clear delineation from the time value of a business enterprise are relevant economic problems (GRUBBSTROM).

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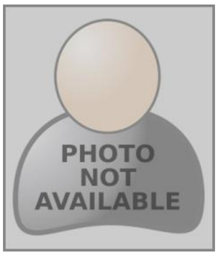

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